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Different decoherence rates of an electron in a multi-state system induced by measurement

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ABSTRACT

We study the different decoherence rates of a multi-state system, consisting of a single electron in three coupled wells, induced by a measurement at arbitrary voltage and temperature. Comparing the effects of the voltage to those of the temperature on dephasing, we show that the measurement voltage in certain sense plays a role of an effective temperature. However, we find that decoherence rates between the different eigenstates are different on the conditions of same voltage and temperature. To further comprehend the dephasing behaviors, we obtain the formulas of the decoherence rates and numerically show that the decoherence rates are influenced by the measurement voltage and temperature. In addition, in the large voltage and temperature regime, the effects of temperature and voltage on decoherence rates are different.

1. Introduction

In recent years, the measurement problem in solid-state systems [1–9,15,16] has attracted wide spread attention due to the development in experimental techniques of mesoscopic structures [10–13]. In particular, one experiment was implemented to measure occupation and spin of a single electron confined in a semiconductor quantum dot using a quantum point contact (QPC) [14]. This experiment demonstrated that the QPC is used as a charge detector with a resolution much better than a single electron charge and a shorter measurement timescale.

With the advent of Nano-scale fabrication and low-temperature measuring techniques, it has become possible to study directly the interaction between coherence and dephasing of electrons in mesoscopic systems. The which-path experiments [10,11] was the first to investigate controllable dephasing with mesoscopic electron structures. Controllable decoherence was realized in a device with a quantum dot embedded in an AB ring and capacitively coupled to a QPC. The detection process led to dephasing of electron states in the quantum dot. From the basics of quantum mechanics, the origin of dephasing is distinguishing the superposition components of a quantum state. The experiments verified the fundamental principle of quantum mechanics. In the controllable dephasing experiments [17–19], environment fluctuations induce dephasing, where a detector played a role of an environment influencing electron states. In this aspect, theory research [1] proved that the continuous detection of one of the coherent superposition states can induce decoherence. The Bloch equation approach developed in Ref. [1] is valid at large voltage regime. In Ref. [20], the measurement of a qubit is studied at arbitrary voltage and temperature.

In Refs. [21,22], a treatment being applicable to measurement at arbitrary measurement voltage and temperature was given. Based on the treatment, we studied the quantum measurement of a multi-state system at zero temperature in our previous work [23]. In Ref. [23], dephasing behavior in the quantum-well-state representation in our model was compared with that in Ref. [24]. It was shown that stronger observation leads to faster dephasing, and the off-diagonal density matrix is not zero in the quantum-well-state basis, where the coherence between the quantum-well-states originates from the superposition components of the same eigenenergy state. Regardless of much research work about decoherence, its mechanism has not been clarified sufficiently [25,26]. For example, the decoherence depends on representation [23]. After measurement, the off-diagonal density matrix is not zero in well-state basis, however, it decays to zero in eigenstate basis. Therefore, it may be important for quantum information to make the mechanism of the decoherence clear.

In this paper, we investigate the different dephasing of a multi-state system in the eigenstate representation induced by a measurement. In the absence of the detector, quantum coherence of the multi-state system is indicated by the oscillations of an electron in three coupled wells. These oscillations stem from the interference between the probability amplitudes of finding an electron in different wells. In the presence of the detector, the measurement destroys these oscillations, i.e., the measurement generates dephasing. Our simulations show that the dephasing rate becomes greater with the increase of the measurement

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voltage and temperature. Though the measurement voltage and temperature have similar effects on dephasing, the decoherence rates between the different eigenstates are different at the same voltage and temperature. In addition, in the large voltage and temperature limit, the impact of voltage and temperature on dephasing is distinct.

2. Model description and formalism

As schematically shown in Fig. 1, we consider an electron in a one-dimensional array of three coupled quantum wells, each of which has a local energy level $\epsilon_j (j=a,b,c)$, measured by a mesoscopic QPC placed near the first well. Here μ_L and μ_R stand for the chemical potentials in the left and right reservoirs of the QPC, respectively. The QPC is operated as a charge detector in order to distinguish which well the electron occupies. The transmission coefficient of the QPC relies on the electron charge state of the quantum well, which changes the electrostatic potential in its vicinity, including the surrounding region of the QPC, therefore the current through the QPC is sensitive to the electrostatic changes. According to the current changes, we can determine which well the electron resides on. The entire system Hamiltonian reads

$$H=H_0+H', \tag{1}$$

$$H_0=H_S+H_{\rm res}, \tag{2}$$

$$H_S=\sum_{j=a}^{c}\epsilon_j c_j^\dagger c_j+\sum_{j=a}^{b}(\Omega_j c_{j+1}^\dagger c_j+{\rm H.c.}), \tag{3}$$

$$H_{\rm res}=\sum_{k,q}(\epsilon_k^L c_k^\dagger c_k+\epsilon_q^R d_q^\dagger d_q), \tag{4}$$

$$H'=\sum_{k,q}\left(\Omega_{qk}+\sum_{j=a}^{c}\chi_{qk}^j c_j^\dagger c_j\right)c_k^\dagger d_q+{\rm H.c.}, \tag{5}$$

where the total Hamiltonian H contains the interaction Hamiltonian H' and the free part H_0, which includes the measured system Hamiltonian H_S and the QPC reservoirs Hamiltonian $H_{\rm res}$. The operators $c_k^\dagger(c_k)$ and $d_q^\dagger(d_q)$ are the electron creation (annihilation) operators of the left and right reservoirs of the QPC, respectively. The operator $c_j^\dagger(c_j)$ is the creation (annihilation) of an electron in the jth well. For simplicity we assume that each well is coupled only to its nearest neighbors with couplings Ω_j and Ω_{j-1}. The Hamiltonian H' describes electron tunneling through QPC, e.g., from the left reservoir to the right reservoir, with tunneling amplitude $\Omega_{qk}+\sum_{j=a}^{c}\chi_{qk}^j c_j^\dagger c_j$, which generally relies on the measured electron's position. This dependence properly describes the

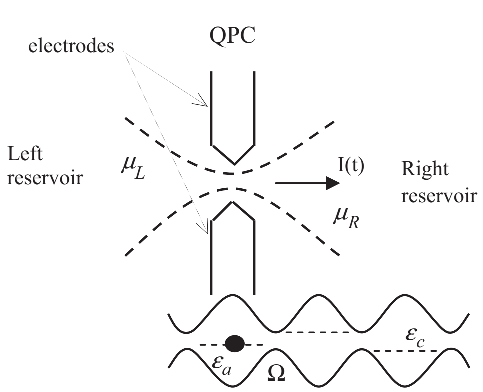

Fig. 1. Schematic picture of three coupled quantum wells subjected to a measurement of a single electron in them using the mesoscopic quantum-point-contact.

correlation between the measured system and the detector. Here we introduce the well state by $|a\rangle$, $|b\rangle$ and $|c\rangle$, corresponding to the electron locating in the jth $(j=a,b,c)$ well. We also introduce the eigenstates $|1\rangle$, $|2\rangle$ and $|3\rangle$, which are the superpositions of the well-states $|a\rangle$, $|b\rangle$ and $|c\rangle$.

Quantum measurement relates to the readout information from the detector and the measurement back-action onto the measured system. The back-action is described by a quantum master equation (QME) satisfied by the reduced density matrix. Regarding the tunneling Hamiltonian H' as perturbation and performing the second-order cumulant expansion, a formal equation for the reduced density matrix is obtained [27]

$$\dot\rho(t)=-i\mathcal{L}\rho(t)-\int_0^t d\tau\langle\mathcal{L}'(t)\mathcal{G}(t,\tau)\mathcal{L}'(\tau)\mathcal{G}^\dagger(t,\tau)\rangle\rho(t), \tag{6}$$

where the Liouvillian superoperators are defined as $\mathcal{L}(\cdots)\equiv[H_S,(\cdots)]$, $\mathcal{L}'(\cdots)\equiv[H',(\cdots)]$, and $\mathcal{G}(t,\tau)(\cdots)\equiv G(t,\tau)(\cdots)G^\dagger(t,\tau)$ with $G(t,\tau)$ the usual propagator (Green's function) associated with H_S. The average $\langle\cdots\rangle$ means tracing over the microscopic degrees of freedom of the detector (or environment). The reduced density matrix $\rho(t)={\rm Tr}_D[\rho_T(t)]$ denotes tracing out all the detector degrees of freedom from the entire density matrix. Following Ref. [22], the measurement back-action can be described by the un-conditional master equation satisfied by the reduced density matrix. The equation is obtained as

$$\dot\rho=-i\mathcal{L}\rho-\tfrac{1}{2}[Q,\tilde{Q}\rho-\rho\tilde{Q}^\dagger], \tag{7}$$

where $Q\equiv\Omega_0+\sum_{j=a}^{c}\chi_j c_j^\dagger c_j$, $\tilde{Q}=\tilde{Q}^{(+)}+\tilde{Q}^{(-)}$, and $\tilde{Q}^{(\pm)}=\tilde{C}^{(\pm)}(\mathcal{L})Q$, with $\tilde{C}^{(\pm)}(\mathcal{L})$ the spectral function of the QPC reservoirs. For simplicity, we have assumed $\Omega_{qk}=\Omega_0$ and $\chi_{qk}^j=\chi_j$, i.e., the tunneling amplitudes are reservoir-state independent. The term $[\cdots]$ describes the back-action of the detector on the measured system. Under wide-band approximation, $\tilde{C}^{(\pm)}(\mathcal{L})$ can be explicitly carried out as [21,22]

$$\tilde{C}^{(\pm)}(\mathcal{L})=\eta[x/(1-e^{-x/T})]_{x=-\mathcal{L}\mp V}, \tag{8}$$

where $\eta=2\pi g_L g_R$, with $g_{L(R)}$ the density of states of the left (right) reservoir. The meaning of the super-operator function $\tilde{C}^{(\pm)}(\mathcal{L})$ was explained using the matrix elements of $\tilde{Q}^{(\pm)}$ [23].

3. Decoherence induced by measurement

3.1. Dephasing versus temperatures and voltages

In the following we study the measurement-induced decoherence behaviors at arbitrary temperature and voltage. Numerically, suppose the coupling strengths between the nearest-neighbor wells identical, $\Omega_j=\Omega$. For the QPC, we assume $\Omega_{qk}\equiv\Omega_0=\Omega$ and quantum-well-state-dependent tunneling coefficients in terms of $\Omega+\chi_j$, with $\chi_1=0.25\Omega$, $\chi_2=0.20\Omega$, $\chi_3=0.15\Omega$. Therefore, our model corresponds to that the experimenter can distinguish the electron's position in each quantum well. For reasons of calculational convenience, we use the unit system of $\hbar=e=k_B=1$, and some physical quantities are simplified as $eV=V$, $k_BT=T$, $\hbar\Omega=\Omega$.

The coherence between the eigenstates $|i\rangle$ and $|j\rangle$ is described by the off-diagonal density-matrix element ρ_{ij}. Without measurement and being coupled with environment, namely, T=V=0, the coherence is kept, as shown in Fig. 2(a)–(c). For fixed temperature and voltage $T=V=2\Omega$, the dephasing behaviors between the different eigenstates are shown in Fig. 2(a′)–(c′). The coherence denoted by ρ_{12} between $|1\rangle$ and $|2\rangle$ is destroyed fastest, as shown in Fig. 2(a′). Relatively, in Fig. 2(b′)