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Electron Raman scattering of a two-electron quantum dot

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ABSTRACT

Electron Raman scattering of a two-electron quantum dot is studied by means of the exact diagonalization and the second-perturbation approach methods. The differential cross section of Raman scattering is obtained. Effects of the external magnetic field and the confinement strength on the differential cross-section are investigated. The results show that the Raman scattering is an experiment that would allow us to probe the angular momentum transitions of the ground state as the magnetic field is varied.

1. Introduction

Quantum dots (QDs) are very small two-dimensional islands of electrons, which are laterally confinement by an artificial potential [1,2]. Their charge and energy spectra are quantized. Hence, they can be thought of as artificial atoms with the field of nucleus replaced by an imposed external potential. In QDs, the most striking feature is that the correlation and magnetic field effects are greatly enhanced compared with their normal counterparts. This feature makes quantum confined semiconductors very promising for possible device applications in microelectronics, nonlinear optics, and many other fields. In recent years, the experimental study of semiconductor QDs is expanding rapidly [3–6], and electron–electron interaction and correlation effects are shown to be of great importance [7–9] in such systems. In the meantime, a large number of theoretical investigations of electronic structures, the magnetic and optical properties in QDs have been performed to explain the experimental observations. Based on a numerical solution of the Coulomb interaction between electrons, a complex ground state behavior (ground-state transitions) as a function of a magnetic field has been predicted [8,10]. Remarkably, these ground-state transitions for a two-electron QD have been observed experimentally [5].

The electron–electron interaction in a QD has profound influence on the ground-state, which occurs in magnetic field only at certain magic values of the total angular momentum L and total spin S [11]. In 1993, Eric Yang et al. investigated the phase diagrams [12], where the most important finding is the discovery of the transition of the quantum numbers L and S of the ground-state in accord with the variation of the magnetic field strength. This fact definitely implies a phase transition, i.e., a transition of structures. Thereby, when the magnetic field continuously increases, the jump of a number of physical properties (such as optical properties [13], electronic heat capacity [11], and magnetization [8]) from a platform to another platform will occur.

Optical methods are convenient experimental tools for studying the properties of low-dimensional semiconductor systems. Electron Raman scattering for its convenience and no damage to materials being measured has become a powerful tool to provide the direct information on the electronic structure and optical properties of low-dimensional semiconductor systems [14]. Recently, electron Raman scattering has been used to investigate the electron–electron interactions of a two-dimensional electron gas by leading experimental groups [15–20]. The calculation of the differential cross-section (DCS) of Raman scattering remains a rather interesting and fundamental issue to achieve a better understanding of the semiconductor nanostructures. Hence, in order to interpret experimental results, many theoretical works usually focus on the calculation of the DCS for Raman scattering [21–23]. On the other hand, some authors have investigated theoretically the electronic Raman scattering of QDs [24–27]. They found that the DCS of QDs strongly depend on the dot size and the strength of the external field. However, only few works were related to the electron Raman scattering of a two-electron QD system so far. Recently, Banerjee et al. investigated the singlet–triplet transition phenomenon of a two-electron QD through Raman spectroscopy [28]. In this work, we will study the electron Raman scattering of a two-electron QD and investigate the influences of the external magnetic field and the confinement strength on the DCS. We will show that the Raman scattering is an

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experiment that would allow us to probe the angular momentum transitions of the ground state as the magnetic field is varied.

2. Theory

Let us first consider a system of two electrons moving in a QD subjected to a parabolic confinement $V(r)=\frac{1}{2}m_e\omega_0^2r^2$, where m_e is the effective mass of an electron, and ω_0 is the characteristic frequency of the confinement potential. The external magnetic field is assumed to be along the z direction. The electron–electron interaction is taken to be the unscreened Coulomb potential. With symmetric gauge for the magnetic field $\vec{A}=(B/2)(-y,x,0)$, the Hamiltonian takes the form

$$H=\sum_{i=1}^{2}\left(\frac{p_i^2}{2m_e}+\frac{1}{2}m_e\omega^2r_i^2\right)+\frac{e^2}{\epsilon r_{12}}-\frac{1}{2}\omega_cL_z-g^*\mu_BBS_z, \tag{1}$$

where $\vec{r}_i$ ($\vec{p}_i$) is the position vector (the momentum vector) of the ith electron originating from the center of the dot; $r_{12}=|\vec{r}_1-\vec{r}_2|$ is the electron–electron separation; ϵ is the effective dielectric constant, g^* is the effective Lande factor; and μ_B is the Bohr magneton. The z component of the total orbital (spin) angular momentum is denoted by L_z (S_z). The frequency of the effective parabolic confinement is given by $\omega=\sqrt{\omega_0^2+\omega_c^2/4}$, where $\omega_c=eB/cm_e$ is the cyclotron frequency.

Introducing the center-of-mass coordinate $\vec{R}=\frac{1}{2}(\vec{r}_1+\vec{r}_2)$ and the relative coordinate $\vec{r}=\vec{r}_2-\vec{r}_1$, the Hamiltonian can be rewritten as

$$H=\frac{P^2}{2\mu_R}+\frac{1}{2}\mu_R\omega^2R^2+\frac{p_r^2}{2\mu_r}+\frac{1}{2}\mu_r\omega^2r^2+\frac{e^2}{\epsilon r}-\frac{1}{2}\omega_cL_z-g^*\mu_BBS_z, \tag{2}$$

where $\mu_R=2m_e$ is the total mass, $\mu_r=m_e/2$ is the reduced mass associated with the relative coordinate.

The Hamiltonian has a cylindrical symmetry which implies that the total orbital angular momentum L is a conserved quantity, i.e., a good quantum number. The total spin of two electrons, S, also is a conserved quantity. Hence, the eigenstates of the two electrons in the two-dimensional cylindrical symmetry QDs can be classified according to the total orbital angular and the total spin momenta of two electrons. In order to obtain the eigenfunctions and the eigenenergies of this system, we diagonalize H in a model space spanned by the translational invariant harmonic product bases $\Phi_{[K]}=\tilde{A}\{[\phi_{n_1\ell_1}^{\omega'}(\vec{R})\phi_{n_2\ell_2}^{\omega'}(\vec{r})]_L\chi_S\}$, where $\chi_S=[\zeta(1)\zeta(2)]_S$, $\zeta(i)$ is the spin state of the ith electron and the spins of two electrons are coupled to S, $\phi_{n\ell}^{\omega'}$ is a two-dimensional harmonic oscillator state with frequency ω' (ω' is considered as an adjustable variational parameter), an energy $(2n+|\ell|+1)\hbar\omega'$, and $\tilde{A}$ is the antisymmetrizer. $[K]$ denotes the whole set of quantum numbers (n_1,ℓ_1,n_2,ℓ_2) in brevity, $\ell_1+\ell_2=L$ is the total angular momentum. The angular momentum $L=$ odd if the spin $S=1$, and $L=$even if $S=0$ such that the wave function is antisymmetrized. The exact diagonalization method consists in spanning the Hamiltonian for a given basis and extracting the lowest eigenvalues of the matrix generated.

The DCS of electron Raman scattering of a volume V per unit solid angle $d\Omega$ for the incoming light with the frequency ω_l and the scattering light with the frequency ω_s is given by Ref. [23]

$$\frac{d^2\sigma}{d\Omega\, d\omega_s}=\frac{V^2\omega_s^2\eta(\omega_s)}{4\pi^2c^4\hbar\eta(\omega_l)}\sum_f|M_{e_1}+M_{e_2}|^2\delta(E_f-E_i), \tag{3}$$

where

$$M_j=\sum_a\frac{\langle f|H_{js}|a\rangle\langle a|H_{jl}|i\rangle}{E_i-E_a+i\Gamma_a},\quad j=1,2, \tag{4}$$

and

$$\delta(E_f-E_i)=\frac{\Gamma_f}{\pi[(E_f-E_i)^2+\Gamma_f^2]}. \tag{5}$$

here $\eta(\omega)$ is the refraction index as a function of the radiation frequency, $|i\rangle$ and $|f\rangle$ denote the initial and final states of the system with their corresponding energies E_i and E_f. $|a\rangle$ is the intermediate state with energy E_a. Γ_a is the life-time width. The dipole operator is independent of the electron spin. The dipole-allowed optical transitions of a two-electron QD are always from the same spin states, but the angular momenta must differ by one unit [29]. Hence, the selection rules determine the intermediate and final states of two-electron systems after the transition. In this work, we restrict our study to the transitions of the $S=0$ states

$$H_{jl}=\frac{|e|}{m_0}\sqrt{\frac{2\pi\hbar}{V\omega_l}}\vec{e}_{jl}\cdot\vec{p}_j,\quad \vec{p}_j=-i\hbar\nabla_j, \tag{6}$$

where m_0 is the free electron mass. The Hamiltonian above H_{jl} describes the interaction with the incident radiation field in the dipole approximation. The Hamiltonian of the interaction with the secondary radiation field is given by

$$H_{js}=\frac{|e|}{m_e}\sqrt{\frac{2\pi\hbar}{V\omega_s}}\vec{e}_{js}\cdot\vec{p}_j,\quad \vec{p}_j=-i\hbar\nabla_j, \tag{7}$$

where $\vec{e}_{jl}$ ($\vec{e}_{js}$) is the unit polarization vector for the incident (secondary) radiation.

3. Results and discussion

In this study, the numerical calculations are carried out on a typical GaAs QD. We have used the following parameters in the calculations: $m_e=0.067m_0$, where m_0 is the free electron mass, $\epsilon=12.53$, and $g^*=0.44$. The lifetimes of the final and intermediate states are $\Gamma_a=\Gamma_f=1$ meV. To see intuitively the feature of low-lying states, we plotted in Fig. 1 the energy spectra of low-lying states with $L\le 2$ and $S=0$ in a two-electron QD as a function of the external magnetic field strength B for two different values of the confinement frequency: (a) $\omega_0=5.0\times10^{13}$ s^{-1} and (b) $\omega_0=7.0\times10^{13}$ s^{-1}. As illustrated in Fig. 1, an important aspect of the magnetic field effects is the changes of the level ordering and then the crossover of two levels can appear. We know that it is the competition between the single particle energy and the interacting energy that finally determines the total energy. The slope of the rising curve depends on the angular momentum L. A smaller L would lead to a larger slope because the negative term $\omega_cL_z/2$ is weaker. Hence, when the external magnetic field increases, the curve with a small L crosses the curve with a larger L because the former is rising faster. Obviously, the crossing would lead to an angular momentum transition of the ground state from one to another. However, the transition is strictly limited to between two magic numbers of L [29]. In Fig. 1, we note the angular momentum transitions of the ground state (i.e., from $L=0$ state to $L=2$ state) and the transition point shifts higher magnetic field with increasing the confinement frequency. As we can see from Fig. 1 that with increasing the magnetic field, the $L=1$ level crosses the $L=2$ level. After the cross point, the energy of the $L=2$ state is lower than that of the $L=1$ state. With further increase in the magnetic field, the $L=0$ level crosses the $L=2$ level (i.e., the ground state transition). After the point of the ground-state transition, the $L=0$ level falls below the $L=0$ level with increasing B. Hence, one can observe the changes of the level ordering of a two-electron QD by electron Raman scattering of athree-level system. We can take the initial state as $L=0$, and consider Raman excitations from this state. In