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Short-pulse heating and analytical solution to non-equilibrium heating process

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ABSTRACT

Analytical solution for lattice and electron temperatures is presented for non-equilibrium energy transfer in gold substrate subjected to a time exponentially decaying short-pulse heating. The finite Lie point symmetries and similarity solution are incorporated in the analysis. Electron temperature distribution obtained from the analytical solutions is compared to its counterpart predicted from the numerical simulations. It is found that the rate of electron temperature decay in the surface region is high, which in turn, results in high rate of lattice site temperature increase due to the collisional energy transfer from the electron sub-system to the lattice-sub system.

1. Introduction

Short pulse of heating of metallic surfaces results in thermal separation of the electron and the lattice sub-systems. Thermal communication in between both sub-systems gives rise to non-equilibrium energy transport in the heated region. The collisional process taking place between excited electrons and the lattice sub-system governs the energy transfer from the electron sub-system to the lattice sub-system. This process continues until the thermal equilibrium is established between the sub-systems. When the heating duration is comparable to electron relaxation time, non-equilibrium energy transfer takes place through the collisional process while dominating over the diffusional energy transfer in the solid. In this case, the Fourier heating model fails to describe the physical insight into heat transfer in the substrate material. Consequently, the electron kinetic theory approach incorporating the electron–lattice site collisions between the lattice and electrons sub-systems becomes essential to account for the formulation energy transport in the solids. Moreover, the closed form solution for the governing equation of the physical problem becomes fruitful, since it provides the functional relation between the independent variables, such as time and space, and the dependent variable, such as temperature. Although the analytical approach giving the approximate solution for the physical problem is obtained earlier [1], the solution presented is limited in time and space scales due to the assumptions made in the analysis. Consequently the general form of the analytical solution for the non-equilibrium energy transport in the metallic substrates due to short-pulse heating becomes essential.

Considerable research studies were carried out to examine short-pulse heating of metallic surfaces. The effect of femtosecond laser pulse trains on optical characteristics and non-equilibrium heat transfer in thin metal films was studied by Sim et al. [2]. They assumed that the electron–electron and electron–phonon collision frequencies vary significantly with the number of pulses per train and the separation time per pulse. Non-equilibrium modeling of heat transfer in a powder layer subjected to a short-pulse heating was presented by Zhang [3]. The findings revealed that the non-equilibrium transport was significant when the pulse length was of less than nanosecond duration. Laser short-pulse heating of metal surface was modeled using molecular dynamics by Zhigilei et al. [4]. They presented the condition leading to photomechanical lamination of single or multiple layers. The contribution of the D-band electrons to ballistic electron transport and interfacial scattering in energy transfer in thin metal films was investigated by Hopkins [5]. The ballistic component of electron transport, leading to electron-interfacial scattering was studied through a ballistic-diffusion approximation of the Boltzmann transport equation. The non-equilibrium properties of electrical and thermal currents in metals were investigated by Huettner [6]. The results indicated that Ohm's law remained a good approximation in most cases whereas the Fourier equation should be supplemented by a relaxation term leading to the hyperbolic heat conduction equation. The generation of acoustic oscillation by short pulses in metals was studied by Afanasiev et al. [7]. They showed that heating by electrons due to the inverse bremsstrahlung absorption by high-power short-pulse laser radiation resulted in parametric generation of non-acrostic waves. In addition Cherenkov generation of non-equilibrium phonons resulted in suppression of the electrons

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heat flux and rapid solidification of the metal lattice. The effect of electron phonon energy exchange on thermal pulse propagation in semiconductors was examined by Gonzalez and Gurevich [8]. The transient temperature of quasi-particle systems was calculated self-consistently through incorporating the electron–phonon energy interactions. Al-Qahtani and Yilbas [9], Al-Theeb and Yilbas [10] and Yilbas et al. [11] studied laser short-pulse heating of metallic surfaces after incorporating the electron kinetic theory approach. The findings revealed that electrons and the lattice sub-system had different temperatures due to the presence of non-equilibrium energy transfer in the laser irradiated region. However, the model studies were involved with the numerical solution of governing transport equations.

The analytical solutions for hyperbolic heat conduction equation were presented earlier [9–11] and the main feature was the solution of the Cattaneo equation with the appropriate boundary conditions. However, short-pulse heating of metallic surfaces is involved with non-equilibrium energy transfer in the lattice and the electrons sub-systems. This requires the consideration of the electron and the lattice sub-systems in the analysis. The approximate solution of the two equation model was presented by Pakdemirli and Yilbas [1] using the perturbation method. However the analytical solution for electron and lattice temperatures provided reasonable accurate solutions for the limited time and space scales. This was because of the approximations made for the first and zero order solutions. Therefore, an extension of the previous study [1] is necessary for extended time and space scales. Consequently, in the present study the closed form solution for electron and lattice temperatures is presented incorporating the Lie point symmetries and similarity solution. The kinetic theory approach is introduced to obtain the governing differential equations for non-equilibrium energy transfer between the electron and the lattice sub-systems. The governing equations are non-dimensionalized prior to the closed form solution.

2. Mathematical analysis

The electron kinetic theory approach considers the electron–phonon collision mechanism through which the energy exchange between the electrons and lattice site atoms occurs. The mathematical model pertinent to electron kinetic approach is given in the previous study [12,13]; hence, the resulting equation is given here, i.e.

$$\left[\left(1+\tau_s\frac{\partial}{\partial\bar{t}}\right)-\frac{\lambda^2}{f}\frac{\partial^2}{\partial\bar{x}^2}\right]\rho C_p\frac{\partial T_L}{\partial\bar{t}}=k\frac{\partial^2 T_L}{\partial\bar{x}^2}+I_o\delta\exp(-\delta|\bar{x}|) \tag{1}$$

When Eq. (1) is decomposed into two equations, the resulting differential equations can be written as:

$$A\frac{\partial T_E}{\partial\bar{t}}=B\frac{\partial^2 T_E}{\partial\bar{x}^2}-C[T_E-T_L]+I_o\delta\exp(-\delta|\bar{x}|)$$
$$D\frac{\partial T_L}{\partial\bar{t}}=C[T_E-T_L] \tag{2}$$

The coefficients A–D can be calculated, i.e.

$$A=\frac{fk\tau_s}{\lambda^2}$$
$$B=k$$
$$C=\frac{fk}{\lambda^2}\left(1-\frac{fk\tau_s}{\rho C_p\lambda^2}\right)$$
$$D=\rho C_p-\frac{fk\tau_s}{\lambda^2}$$

where ρ, k, C_p, f, and λ are density, thermal conductivity, specific heat, fraction of electron energy transfer during a single collision, and lattice mean free path, respectively. Eq. (2) is identical to the equations given in the two-equation model, since the governing equations in the two-equation model are [14]

$$C_e\frac{\partial T_E}{\partial\bar{t}}=\nabla(kT_E)-G[T_E-T_L]+S$$
$$C_L\frac{\partial T_L}{\partial\bar{t}}=G[T_E-T_L] \tag{3}$$

where T_E and T_L are the electron and lattice site temperatures, S is the laser source term, and C_E and C_L are the electron and lattice heat capacities, respectively. G is the electron–phonon coupling factor, given by

$$G=\frac{\pi^2 m_e N\overline{V}^2}{6\tau_p T_E}$$

where m_e, N, $\overline{V}$, and τ_p are electron mass, electron number density, electron drift velocity and the electron mean free time between electron–phonon coupling respectively. Consequently, setting the coefficients of Eqs. (2) and (3), it yields

$$\frac{fk\tau_s}{\lambda^2}=C_e$$
$$\frac{fk}{\lambda^2}\left(1-\frac{fk\tau_s}{\rho C_p\lambda^2}\right)=G$$
$$\rho C_p-\frac{fk\tau_s}{\lambda^2}=C_l$$

where $\tau_s=C_E/G$ and $C_E=\gamma T_e$, where γ is constant [15].

Moreover, the energy transport equations for electron and lattice subsystems without source term can be written as [14]

$$C_E\frac{\partial T_E}{\partial\bar{t}}=k\frac{\partial^2 T_E}{\partial\bar{x}^2}-G(T_E-T_L),$$
$$C_L\frac{\partial T_L}{\partial\bar{t}}=G(T_E-T_L). \tag{4}$$

where T_E and T_L are the electron and lattice site temperatures respectively. C_E is the electron heat capacity, C_L is the lattice heat capacity, G is the electron–phonon coupling factor and k is the thermal conductivity. $\bar{x}$ is the dimensional lattice depth and $\bar{t}$ is the dimensional time.

If the dimensionless quantities are defined as

$$\theta_E=\frac{T_E}{T_o},\quad \theta_L=\frac{T_L}{T_o},\quad x=\bar{x}d,\quad t=\frac{\bar{t}}{C_E/G} \tag{5}$$

Eqs. (1) and (2) reduce to the following form:

$$\frac{\partial\theta_E}{\partial t}=\alpha\frac{\partial^2\theta_E}{\partial x^2}-(\theta_E-\theta_L)$$
$$\frac{\partial\theta_L}{\partial t}=\varepsilon(\theta_E-\theta_L) \tag{6}$$

where

$$\alpha=\frac{kd^2}{G},\quad \varepsilon=\frac{C_E}{C_L}$$

In the above formulation, $d=1/\lambda$ and λ is the mean free path of electrons. The model admits the following Lie point symmetry [1]:

$$X=a\frac{\partial}{\partial x}+b\frac{\partial}{\partial t}+(c\theta_E+f(x,t))\frac{\partial}{\partial\theta_E}+(c\theta_L+g(x,t))\frac{\partial}{\partial\theta_L} \tag{7}$$

Parameters a–c represent finite Lie point symmetries, whereas $(\theta_E,\theta_L)=(f,g)$ is an arbitrary solution for the system [15]. A similarity solution can be constructed using the parameter b and selecting $a=1$ and $c=0$.

So we can get a reduced system by using the symmetry generator $Y=\partial/\partial x+b(\partial/\partial t)$, where the generator Y has a canonical