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Broadband characteristics of vibration energy harvesting using one-dimensional phononic piezoelectric cantilever beams

Zhongsheng Chen *, Yongmin Yang, Zhimiao Lu, Yanting Luo

Key Laboratory of Science and Technology on Integrated Logistics Support, College of Mechatronic Engineering and Automation, National University of Defense Technology, Changsha, Hunan 410073, PR China

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ABSTRACT

Nowadays broadband vibration energy harvesting using piezoelectric effect has become a research hotspot. The innovation in this paper is the widening of the resonant bandwidth of a piezoelectric harvester based on phononic band gaps, which is called one-dimensional phononic piezoelectric cantilever beams (PPCBs). Broadband characteristics of one-dimensional PPCBs are analyzed deeply and the vibration band gap can be calculated. The effects of different parameters on the vibration band gap are presented by both numerical and finite element simulations. Finally experimental tests are conducted to validate the proposed method. It can be concluded that it is feasible to use the PPCB for broadband vibration energy harvesting and there should be a compromise among related parameters for low-frequency vibrations.

1. Introduction

Power consumptions of wireless sensor nodes are becoming smaller and smaller with the development of advanced micro-electronic technologies [1]. Thus energy harvesting has been looked at as a promising way for independent power supplies for micro-sensors in the future, which often converts ambient energy (e.g. wind, thermal and vibration) into electric energy. In particular, piezoelectric vibration energy harvesting (VEH) has some unique advantages of large force-electric coupling coefficient and power density, less electromagnetic inference and easy integration, and so it has been studied widely [2–4].

Classical piezoelectric VEH methods are mostly based on cantilever beam configurations and second-order linear equations are used to describe their dynamical characteristics. Studies have exposed that their power outputs will reach the peak values only at the resonant frequency. Otherwise, they will drop dramatically [3,4]. As we all know, the main natural frequency of a linear piezoelectric cantilever beam is just a single frequency, and so its resonant bandwidth is very small. However, in the vast majority of cases the ambient vibrations have their energy distributed over a wide spectrum of frequencies, with significant predominance of low frequency components (e.g. 20 Hz–200 Hz). Linear VEH devices are difficult to match broadband vibrations and their efficiencies will always be low. To date, many methods have been studied to solve this problem and they can be divided into two classes. The first one is to adjust the resonant frequency of a single harvester so that it can match the main frequency of the ambient vibration at all times. Peters et al. [5] proposed a tunable resonator by mechanical stiffening of the structure using piezoelectric actuators. The other one is to design a broadband harvester directly. Typically nonlinear piezoelectric VEH devices have been considered as a promising way of broadband VEH in recent years [6–9]. By now, these two methods have been testified by both analytical and experimental results.

Phononic crystals (PCs) have become a research hot-spot in condensed matter physics fields recently. PCs are periodic structures composed of at least two classes of elastic material. A peculiar feature of PCs is called as 'band gaps'. Band gaps are defined as frequency intervals where elastic waves are forbidden from propagating. The propagation characteristics of elastic waves can be controlled by artificially designing the structures of PCs. Thus PCs possess rich novel physical properties and promising applications [10–12]. In particular, band gaps in periodic elastic structures can make them extremely appealing as broad mechanical filters. Then energy in vibrations will localize in the form of an oscillatory motion of the internal structural elements, so that the piezoelectric effect can be exploited to convert the localized vibration energy into electrical energy. Interestingly, this idea is exactly consistent with the need for

* Corresponding author. Tel.: +86 731 84573394; fax: +86 731 84573361.
E-mail address: czs_study@sina.com (Z. Chen).

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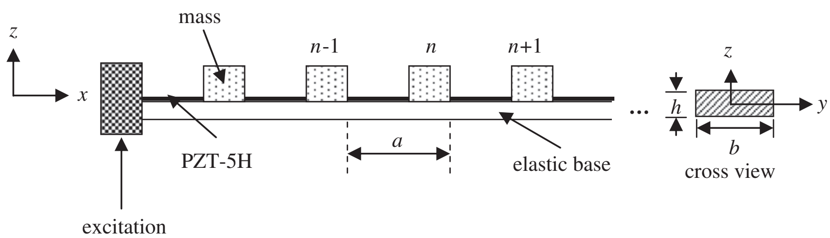

Fig. 1. Diagram of a one-dimensional PPCB.

broadband VEH, and so PCs can be used for broadband VEH. Wu et al. [13] studied noise energy harvesting based on PCs and testified its feasibility by experimental results. However, little work has been done on broadband VEH based on PCs. The novelty of this paper is to deeply study the broadband characteristics of VEH using one-dimensional phononic piezoelectric cantilever beams (PPCBs). Effects of different parameters on the bandwidth of one-dimension PPCBs are analyzed by theoretical analysis and experimental results. Our main motivation is to provide theoretical guidelines for optimal designing of one-dimensional PPCBs in engineering applications.

2. Theoretical analysis on the bandwidth of a one-dimensional PPCB

2.1. Mechanisms statement

The diagram of a one-dimensional PPCB is shown in Fig. 1. It can be built by sticking PZT patches and metal masses on the elastic base periodically. The basic element of the PPCB is called as 'unit cell' and its length is denoted as a. The weight of each mass is denoted as m. Here PZT-5H piezoelectric material is used.

Mechanical vibrations can be physically looked at as a special kind of elastic waves. Vibrations will propagate along the one-dimensional PPCB when excited. Then an energy band will be generated due to its periodic structure, where the frequency range without dispersive curves is named as band gap or vibration band gap [9]. Vibrations falling in the band gap will be forbidden to propagate. Also vibration energy cannot be dissipated, so they have to localize in some unit cells of the one-dimensional PPCB. Thus PZT patches on these cells will become good energy absorbers and can be used to convert localized vibration energy into electrical energy based on piezoelectric effects. In this way, broadband VEH can be achieved. In this sense, vibration band gaps of one-dimensional PPCBs can be looked at as resonant bandwidths of VEHs.

2.2. Analytical calculation of vibration band gaps

In this section the transfer matrix method is used to calculate vibration band gaps of one-dimensional PPCBs [14]. The horizontal and vertical axes in Fig. 1 are defined as the x-axis and z-axis respectively. The vertical displacement of each mass along the x-axis is denoted as $u(x,t)$. When a is far larger than the size of each mass (e.g. at least five times), each unit cell in the one-dimensional PPCB can be looked as an Euler–Bernoulli beam. Then the flexural elastic wave propagation equation along the x-axis can be written as

$$EI\frac{\partial^4 u(x,t)}{\partial x^4}+\rho A\frac{\partial^2 u(x,t)}{\partial t^2}=0 \tag{1}$$

where ρ—the equivalent density, E—the equivalent elastic modulus, A—the cross sectional area of the beam ($A=b\times h$), and I—the moment of inertia ($I=bh^2/12$).

Denoting $u(x,t)=Y(t)\varphi(x)$ based on the separated-variable method, Eq. (1) can be transformed into Eq. (2).

$$m\ddot{Y}(t)\varphi(x)+EI\varphi^{(4)}(x)Y(t)=0 \tag{2}$$

Furthermore, we have

$$\frac{\varphi^{(4)}(x)}{\varphi(x)}=-\frac{m}{EI}\frac{\ddot{Y}(t)}{Y(t)} \tag{3}$$

Obviously, the left part of Eq. (3) is just the function of x, while the right part is just the function of t. Thus Eq. (3) will be valid only when both parts are equal to the same constant, i.e.,

$$\frac{\varphi^{(4)}(x)}{\varphi(x)}=-\frac{m}{EI}\frac{\ddot{Y}(t)}{Y(t)}=\Delta^4 \tag{4}$$

where Δ is a constant.

Denoting $\omega=\Delta^2\sqrt{EI/m}$, Eq. (4) can be transformed into Eq. (5).

$$\begin{cases}\varphi^{(4)}(x)-\Delta^4\varphi(x)=0\\ \ddot{Y}(t)+\omega^2 Y(t)=0\end{cases} \tag{5}$$

The solution of the second equation in Eq. (5) can be written as

$$Y(t)=A_1\cos\omega t+A_2\sin\omega t \tag{6}$$

where the coefficients of A_1, A_2 can be calculated using initial conditions.

Let $\varphi(x)=\beta e^{\lambda x}$, we can derive the eigenfunction of Eq. (5) as $\lambda^4=\Delta^4$. Its four roots are $\lambda=\pm\Delta$, $\lambda=\pm j\Delta$. Thus the general solution of $\varphi(x)$ can be expressed as follows:

$$\varphi(x)=\beta_1\cos\Delta x+\beta_2\sin\Delta x+\beta_3\cosh\Delta x+\beta_4\sinh\Delta x \tag{7}$$

where the coefficients of β_i ($i=1,2,3,4$) can be calculated by two boundary conditions of each unit cell.

For the nth unit cell, the solution of Eq. (1) can be expressed as Eq. (8) by combining Eq. (6) and Eq. (7).

$$u(x'_n,t)=e^{i\omega t}[A_n^+\cos(\alpha x'_n)+A_n^-\sin(\alpha x'_n)+B_n^+\cosh(\alpha x'_n)+B_n^-\sinh(\alpha x'_n)] \tag{8}$$

Similarly, the solution of Eq. (1) for the $(n+1)$th unit cell can be expressed as Eq. (9).

$$u(x'_{n+1},t)=e^{i\omega t}[A_{n+1}^+\cos(\alpha x'_{n+1})+A_{n+1}^-\sin(\alpha x'_{n+1})+B_{n+1}^+\cosh(\alpha x'_{n+1})+B_{n+1}^-\sinh(\alpha x'_{n+1})] \tag{9}$$

where $\alpha^4=\rho A\omega^2/EI$, $x'_n=x-x_n$, $x'_{n+1}=x-x_{n+1}$. x_n is the x-axis coordinate of the nth unit cell. x_{n+1} is the x-axis coordinate of the $(n+1)$th unit cell. A_n^+, A_n^-, B_n^+, B_n^- are elastic wave parameters in the nth unit.