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Limiting effects of geometrical and optical nonlinearities on the squeezing in optomechanics

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ABSTRACT

In recent experiments, the re-thermalization time of the mechanical resonator is stated as the limiting factor for quantum applications of optomechanical systems. To explain the origin of this limitation, an analytical nonlinear investigation supported by the recent successful experimental laser cooling parameters is carried out in this work. To this end, the effects of geometrical and the optical nonlinearities on the squeezing are studied and are in a good agreement with the experimental results. It appears that highly squeezed state are generated where these nonlinearities are minimized and that high nonlinearities are limiting factors to reach the quantum ground state.

1. Introduction

Squeezing is a beautiful quantum phenomenon with amazing potential applications [1,2] of which the most recent are connected with continuous variables quantum information [3,4] and ultrasensitive measurement of weak perturbations as the gravitational waves [5–7]. Squeezed states are nonclassical states in which the variance of at least one of the canonical variables is reduced below the noise level of zero point fluctuations. To generate squeezed states, the common technique consists to use an optical cavity filled with a nonlinear Kerr medium which is fed with an external pumping field [8]. With the recent advances in cooling techniques for nano scale optomechanical systems, various setups have been designed for quantum ground state engineering of mechanical mirrors with highly squeezed states of light [9–14]. Indeed, with such technique it is now possible to obtain effective phonon number less than 1 [15–20]. The limiting factors to obtain much lower phonon number are the re-thermalization time of the mechanical resonator $\tau_{th} = \hbar Q_m / k_B T$ (where $\hbar$ is Planck's constant, Q_m is the mechanical quality factor, k_B the Boltzmann constant and T is the temperature of the support), which competes with the cooling, and the ubiquitous phase noise of the input laser which can create a discrepancy between experimental results and theoretical prediction [15,16]. Nevertheless, a lot of theoretical studies on the subject has been carried out in the last decade and several proposals have been produced [21–23]. In Ref. [23] we applied the technique of back-action cooling to show that the cooling of the nanomechanical oscillator to its ground state is limited by the effects of both optical and mechanical nonlinearities.

In this paper, by using the parameters of the experimental laser cooling of Ref. [15], we extend the previous treatment to show through analytical study that there are the nonlinearities which limit the squeezing in optomechanics. The first one which depends on the geometry of the mechanical structure is known as the geometrical nonlinearity derives from the nonlinear dynamics of the beams [24–26]. The second one is the optical nonlinearity which appears as a nonlinear phase shift [10]. The geometrical nonlinearity which is always present and not negligible in nano resonators, is shown to be a limiting factor to reach the quantum ground state as suggested in Ref. [26]. In the same way, it is shown that high absolute values of the optical nonlinearity limit the squeezing of the output intensity.

The paper is structured as follows. Section 2 will set the stage for exploring the dynamics of our system, deriving in particular the nonlinear Quantum Langevin Equations and the linearized equations of motion. Sections 3 and 4 subsequently make use of numerical simulations to discuss the squeezing of the mechanical and the optical output quadratures. Finally, we conclude with an outlook of possible future directions.

2. Dynamics equations

We consider an optomechanical resonator described on Fig. 1 of Ref. [15]. The dynamics equations of a mechanical oscillator coupled to a driven cavity are usually derived from a single-mode

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Hamiltonian [12,23,24,27] as

$$\ddot{x}_m + \Gamma_m \dot{x}_m + \Omega_m^2 x_m - \beta'' \Omega_m = g_M \Omega_m \left|\alpha(t)\right|^2 + \frac{F_{th}}{M x_{ZPF}}, \tag{1a}$$

$$\dot{\alpha} = \left[i\left(\Delta + \frac{g_M}{x_{ZPF}} x\right) - \frac{\kappa}{2}\right]\alpha(t) - i\varepsilon^{in} + \sqrt{\kappa}\alpha^{in}, \tag{1b}$$

x_m and p_m are respectively the dimensionless position and momentum operators of the mechanical oscillator related to their counterparts operators of the nanobeam as $x = \sqrt{\hbar/2M\Omega_m} x_m = x_{ZPF} x_m$ and $p = \hbar/x_{ZPF} p_m$, with $[x_m, p_m] = 2i$. The parameters $g_M = \sqrt{2}\omega_c(x_{ZPF}/d_0)$, d_0, Ω_m and ε^{in} are respectively the optomechanical coupling, the cavity length, the mechanical frequency and the amplitude of the input laser beam. The cavity decay rate and the mechanical damping of the mechanical oscillator are respectively represented by κ and Γ_m. The laser-cavity detuning is $\Delta = \omega_\ell - \omega_c$ with ω_c the optical cavity mode frequency and ω_ℓ the laser frequency. The terms F_{th} and α^{in} represent the Langevin force fluctuations and the input laser fluctuations respectively. The terms $\beta'' = (\beta' x_{ZPF}^2 x_m^3)/\Omega_m$ and $g_M x_m \alpha$ represent the mechanical and optical anharmonic terms respectively. When the nanoresonator is subjected to a large displacement amplitudes, it displays a striking nonlinearity β'' in its response. This comes about because the flexure causes the beam to lengthen, which at large amplitudes add a significant correction to the overall elastic response of the beam [26]. The optical anharmonic term is another kind of Kerr medium, which has a mechanical origin: the radiation pressure induces a coupling between the position of the doubly clamped flexural resonator and the phase–intensity of the light beam, thus modifying the opticalpath known as the phase shift.

One can derived from the set of equations (1) the following nonlinear quantum Langevin equations (QLEs) [23]:

$$\dot{x}_m = \Omega_m p_m \tag{2a}$$

$$\dot{p}_m = -\Omega_m x_m - \Gamma_m p_m + g_M \alpha^\dagger \alpha + \beta'' + F_{th} \tag{2b}$$

$$\dot{\alpha} = \left[i(\Delta + g_M x_m) - \frac{\kappa}{2}\right]\alpha - i\varepsilon^{in} + \sqrt{\kappa}\alpha^{in} \tag{2c}$$

$$\dot{\alpha}^\dagger = \left[-i(\Delta + g_M x_m) - \frac{\kappa}{2}\right]\alpha^\dagger + i\varepsilon^* + \sqrt{\kappa}\alpha^{in\dagger}. \tag{2d}$$

By setting the time derivatives to zero in the set of nonlinear equations (2), the stationary values of the position of the oscillator and the amplitude of the cavity field are

$$\overline{x}_m = 2\frac{g_M}{\Omega_m}\left|\overline{\alpha}\right|^2, \quad \left|\overline{\alpha}\right|^2 = \frac{2\kappa P_{in}}{\hbar\omega_\ell\left((\Delta + g_M x_m)^2 + \frac{\kappa^2}{4}\right)}. \tag{3}$$

The values of $\overline{x}$ obey the following third order algebraic equation:

$$\overline{x}^3 + \frac{2\Delta x_{ZPF}}{g_M}\overline{x}^2 + (4\Delta^2 + \kappa^2)\frac{x_{ZPF}^2}{4g_M^2}\overline{x} - \frac{4\kappa x_{ZPF}^3 P_{in}}{\hbar\Omega_m\omega_\ell g_M} = 0. \tag{4}$$

From Eqs. (3) and (4), it appears that both $\overline{\alpha}$ and $\overline{x}_m$ increase when the input laser power P_{in} increases.

Using the experimental parameters of Ref. [15] at the detuning of $\Delta = \Omega_m$ and for $P_{in} = 1$ mW, we obtain the following values of $\overline{x}$ which are in the range of those obtained experimentally in Refs. [14,17]: 1.28×10^{-13}, $-1.09 \times 10^{-8} + 7.43 \times 10^{-10} i$, $-1.09 \times 10^{-8} - 7.43 \times 10^{-10} i$. The first solution, which is real and small, corresponds to the stable regime of the mechanical resonator, while the two conjugate others, which have the same module ($\left|\overline{x}\right| \approx 1.09 \times 10^{-8}$), correspond to the unstable regime.

For $\left|\overline{\alpha}\right| \gg 1$ (satisfied in Ref. [15]), the above QLEs can be linearized by expanding the operators around their steady states: $x_m = \overline{x}_m + \delta x_m$ and $\alpha = \overline{\alpha} + \delta\alpha$. By introducing the vector of quadrature fluctuations $u(t) = (\delta x_m(t), \delta p_m(t), \delta I(t), \delta\varphi(t))^T$ and the vector of noises $n(t) = (0, F_{th}(t), \sqrt{\kappa}\delta I^{in}(t), \sqrt{\kappa}\delta\varphi^{in}(t))^T$, where $\delta I = (\delta\alpha^\dagger + \delta\alpha)$, $\delta\varphi = i(\delta\alpha^\dagger - \delta\alpha)$ are the intracavity quadratures of the intensity and the phase , and the corresponding hermitian input noise operators δI^{in} , $\delta\varphi^{in}$, the linearized dynamics of the system can be written in a compact form

$$\dot{u}(t) = Au(t) + n(t), \tag{5a}$$

with

$$A = \begin{pmatrix} 0 & \Omega_m & 0 & 0 \\ \Omega_m(\beta - 1) & -\Gamma_m & G & 0 \\ 0 & 0 & -\frac{\kappa}{2} & -\tilde{\Delta} \\ G & 0 & \tilde{\Delta} & -\frac{\kappa}{2} \end{pmatrix}. \tag{5b}$$

The higher order of fluctuations are safely neglected. The linearized QLEs show that the mechanical mode is coupled to the cavity mode quadrature fluctuations by the effective optomechanical coupling $G = g_M\left|\overline{\alpha}\right|$, which can be made large by increasing the input laser power P_{in}. $\beta = (3\beta' x_{ZPF}^2 \overline{x}_m^2)/\Omega_m^2$ and $\tilde{\Delta} = \Delta + g_M\overline{x}_m$ denote the dimensionless geometrical nonlinearity and the effective detuning respectively. The range values of the geometrical and optical nonlinearities are given in Table 1. One remarks that β and η increase when $\overline{x}_m$ increases and they reach their maximum values at the detuning $\Delta \approx \Omega_m$. As expected in Table 1, the optical and the mechanical effects are respectively highly pronounced at the optical ($\Delta \approx 0$) and the mechanical ($\Delta \approx \Omega_m$) resonances [12]. This leads us to investigate the squeezing at this particular sidebands.

3. Squeezing of the mechanical quadratures

The dynamics of mechanical fluctuations is obtained by writing Eqs. (5) in the Fourier space

$$B(\Omega)u(\Omega) + n(\Omega) = 0, \tag{6a}$$

where

$$B(\Omega) = \begin{pmatrix} i\Omega & \Omega_m & 0 & 0 \\ \Omega_m(\beta - 1) & (i\Omega - \Gamma_m) & G & 0 \\ 0 & 0 & (i\Omega - \frac{\kappa}{2}) & -\tilde{\Delta} \\ G & 0 & \tilde{\Delta} & (i\Omega - \frac{\kappa}{2}) \end{pmatrix}. \tag{6b}$$

Solving the matrix equation straightforwardly, we obtain the solution for the mechanical displacement operator to be

$$\chi_{eff}^{-1}(\Omega)\delta x_m(\Omega) = a_1 G\Omega_m\sqrt{\kappa}\left(\tilde{\Delta}^2 + \frac{\kappa^2}{4} - \omega^2 + i\kappa\Omega\right) \times \left[-\tilde{\Delta}\delta\varphi^{in} + \left(-i\Omega + \frac{\kappa}{2}\right)\delta I^{in}\right] + \Omega_m F_{th}, \tag{7}$$

where

$$a_1 = \left[\left(\tilde{\Delta}^2 + \frac{\kappa^2}{4} - \Omega^2\right)^2 + \kappa^2\Omega^2\right]^{-1}, \tag{8}$$

Table 1
The range of values of the optical nonlinearity η and the geometrical nonlinearity β at the detuning $\Delta = 0$ and $\Delta = \Omega_m$ respectively, using the parameters of Ref. [15].

Detuning Δ	Mean displacement of the nanobeam $\overline{x}$ (m)	Range of values of nonlinearities
0	2.77×10^{-11}	$\eta \in [2.54 \times 10^{-3}; 6.79 \times 10^{-2}]$
	7.42×10^{-10}	$\beta \in [7.87 \times 10^{-6}; 5.72 \times 10^{-4}]$
Ω_m	1.27×10^{-13}	$\eta \in [1.17 \times 10^{-5}; 1]$
	1.09×10^{-8}	$\beta \in [1.66 \times 10^{-10}; 1.22]$