Physica B 411 (2013) 52–55

Contents lists available at SciVerse ScienceDirect

Physica B

journal homepage: www.elsevier.com/locate/physb

Theoretical study about the relations among coefficients of stimulated emission, spontaneous emission and absorption in indirect bandgap semiconductor

José M. Escalante *, Alejandro Martínez

Nanophotonics Technology Center, Universitat Politècnica de València, 46022 Valencia, Spain

ARTICLE INFO

Article history:
Received 27 June 2012
Received in revised form
3 October 2012
Accepted 20 November 2012
Available online 29 November 2012

Keywords:
Einstein's coefficients
Phonons
Photons
Indirect bandgap semiconductor

ABSTRACT

We show a theoretical study that permits us to obtain the relationship between the coefficients of stimulated emission, spontaneous emission and absorption in indirect bandgap semiconductors, including multiphoton and multiphonon processes.

1. Introduction

In Ref. [1], Einstein introduced the concept of stimulated emission of photons and established the idea of coherent generation of photons (the mechanism behind modern lasers). By postulating some hypotheses on the emission and absorption of radiation, he obtained the known Einstein relations between the coefficients among the coefficients of spontaneous emission, stimulated emission and absorption for atoms or molecules [2].

The general approach of Einstein was to assign rate constant to the three radiative processes appearing, which are given between two energy levels Fig. 1.

In Einstein days, most radiative transitions of interest took place between energy levels of an atom, which are very isolated, sharp energy levels. The carrier density therefore referred to the density of atoms with electrons in either energy level 1 or 2. In the current context of direct bandgap semiconductor, we must interpret these definition somewhat differently because in semiconductor, the energy levels are neither isolated nor sharp [2] (Fig. 2).

To take into account the continuous nature of energy states in the semiconductor that are considering now, we restrict our attention to a differential population of state pairs exiting between E_{21} and $E_{21}+dE_{21}$. The calculus to get the differential expression for different rate transition and obtaining the relation between the Einstein coefficients for this case can be consulted in many manuals about laser theory [2,3], and can be seen that the relationships among the coefficients remains the same.

More recently, Chen et al. presented a theoretical treatment of optical gain in indirect bandgap semiconductors [4]. As a secondary result in Ref. [4], the relationships among Einstein's coefficients (A_{cv} is the spontaneous emission coefficient, B_{cv} is the stimulated emission coefficient, and B_{vc} is the absorption coefficient) for a simple two-level system were obtained. Although these relationships are correct, they do not show the complete landscape for indirect bandgap semiconductors, because the process of stimulated emission of phonons is not considered [5–7].

In this work, we consider the concept of stimulated emission of photons along with the concept of stimulated emission of phonons [5–7] in order to get the complete relationships among those coefficients in indirect bandgap semiconductors. We consider an indirect bandgap semiconductor in which the transition of electrons from the conduction band (CB) to the valence band (VB) is mediated by the emission of both a single-mode phonon and a single-mode photon. We also discuss the similarity between our work and Einstein's result for direct bandgap semiconductors from a physical point of view and the results that were obtained for Chen et al. [4].

2. Einstein's relations for indirect bandgap semiconductor

Our study is based on a simple band structure of indirect bandgap semiconductor as plotted in Fig. 1, with emission of a single phonon and a single photon. It has to be stressed that, from

---

* Corresponding author. Tel.: +34 66 737 70 52; fax: +34 96 387 78 27.
E-mail address: jmescala@ntc.upv.es (J.M. Escalante).

0921-4526/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.physb.2012.11.024

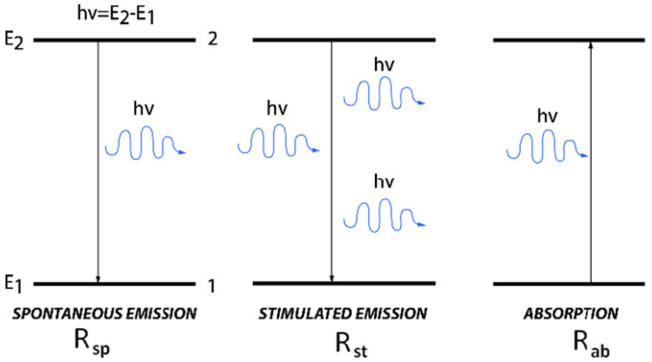

Fig. 1. Transitions rates between two energy levels.

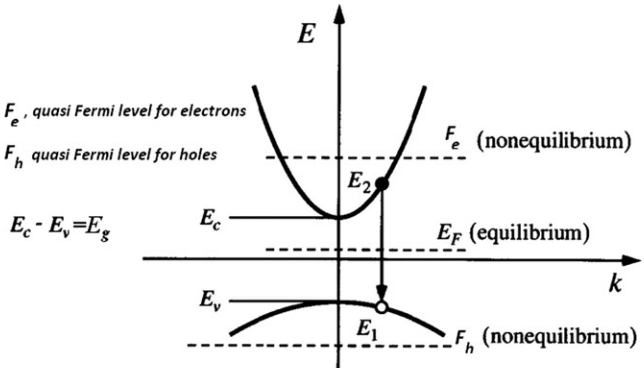

Fig. 2. Energy vs. momentum electronic band structure for direct bandgap semiconductor (E_g, energy gap).

a physical point of view, photons and phonons have a great similarity in some aspects. Both particles are quanta of a classical vibration fields (the photon of the electromagnetic field and the phonon of vibration field of a crystal lattice), they have no mass, their interaction with electrons is very similar (depends on the polarization of the particles) and they are bosons, so both follow the same Bose–Einstein statistics. In this sense, the emission of a photon (phonon) is proportional to the density of photons (phonons) respectively rather than the density of unoccupied states as it occurs for fermions. Therefore, we have to consider the process of the stimulated emission of phonons [5–7], when studying the light emission from an indirect bandgap semiconductor. Following the same idea as in the Einstein's work [1], we will try to get the relationships among different coefficients of emission and absorption in indirect bandgap semiconductors.

We consider an indirect bandgap semiconductor in which the transition of electrons from the CB to the VB is mediated by the emission of both a single-mode phonon and a single-mode photon, as shown in Fig. 1. The rate equations for transition between the CB and the VB by single-mode phonon and single-mode photon emission in indirect bandgap semiconductors in this case are [4–8]

$$R_{kq}=B_{kq}\rho_k\rho_q N_C N_V\frac{\pi}{8}\left(\hbar\omega+\hbar\Omega-E_g\right)^2 e^{\left(-\frac{\hbar\omega+\hbar\Omega-\Delta F}{K_BT}\right)} \tag{1}$$

$$R_{k0}=B_{k0}\rho_k N_C N_V\frac{\pi}{8}\left(\hbar\omega+\hbar\Omega-E_g\right)^2 e^{\left(-\frac{\hbar\omega+\hbar\Omega-\Delta F}{K_BT}\right)} \tag{2}$$

$$R_{0q}=B_{0q}\rho_q N_C N_V\frac{\pi}{8}\left(\hbar\omega+\hbar\Omega-E_g\right)^2 e^{\left(-\frac{\hbar\omega+\hbar\Omega-\Delta F}{K_BT}\right)} \tag{3}$$

$$R_{00}=B_{00} N_C N_V\frac{\pi}{8}\left(\hbar\omega+\hbar\Omega-E_g\right)^2 e^{\left(-\frac{\hbar\omega+\hbar\Omega-\Delta F}{K_BT}\right)} \tag{4}$$

$$R_{ab}=B_{ab}\rho_k\rho_q N_C N_V\frac{\pi}{8}\left(\hbar\omega+\hbar\Omega-E_g\right)^2 \tag{5}$$

where R_{kq} is the rate of stimulated emission of photons and phonons, R_{k0} is the rate of stimulated emission of photons and spontaneous emission of phonons, R_{0q} is the rate of spontaneous emission of photons and stimulated emission of phonons, R_{00} is the rate of spontaneous emission of photons and phonons, R_{ab} is the absorption rate, ρ_k is the photon density per energy interval, ρ_q is the phonon density per energy interval, B_{kq} is the coefficient of stimulated emission of photons and phonons, B_{k0} is the coefficient of stimulated emission of photons and spontaneous emission of phonons, B_{0q} is the coefficient of spontaneous emission of photons and stimulated emission of phonons, B_{00} is the coefficient of spontaneous emission of photons and phonons coefficient, B_{ab} is the absorption coefficient, $N_C=(1/2\pi^2)(2m_e^*/\hbar^2)^{3/2}$, $N_V=(1/2\pi^2)(2m_h^*/\hbar^2)^{3/2}$ (where m_e^* and m_h^* are the effective masses of electrons and holes, respectively), $\hbar\omega$ is the photon energy, $\hbar\Omega$ is the phonon energy, ΔF is the difference between the quasi-Fermi levels for electrons and holes, K_B is the Boltzmann constant, and T is the temperature.

To obtain Eqs. (1)–(5), we have considered effects of spontaneous and stimulated emission of both photons and phonons [5–7]. Although the spontaneous and stimulated emission processes are well known for photons, but not for phonons. The experimental results about the stimulated and spontaneous emission of phonon in indirect bandgap semiconductor as silicon or germanium have not been observed, because the emissions of photons and phonons, both stimulated and spontaneous, in this kind of semiconductors take place simultaneously, so distinguishing the different contributions is quite complicated.

Nevertheless there are different studies which support the idea of stimulated emission of phonons. For instances, in Ref. [6] the researchers present a study about the possible generation of coherent transverse acoustic phonons in the 10^{12}–10^{13} Hz frequency range by indirect photon–phonon interband transition considering the idea of stimulated emission of phonons. In Ref. [8] the researchers establish a model to calculate the optical gain coefficients for indirect bandgap semiconductor which is based on the same idea. In Ref. [9], a similar effect to Purcell effect is observed experimentally which is related with stimulated and spontaneous emission processes. So taking into account these different works, we consider plausible the idea of stimulated emission of phonon in indirect bandgap semiconductor Fig. 3.

To obtain these expressions we considered that the quasi-Fermi level for electrons and holes locate far away from CB edge and VB

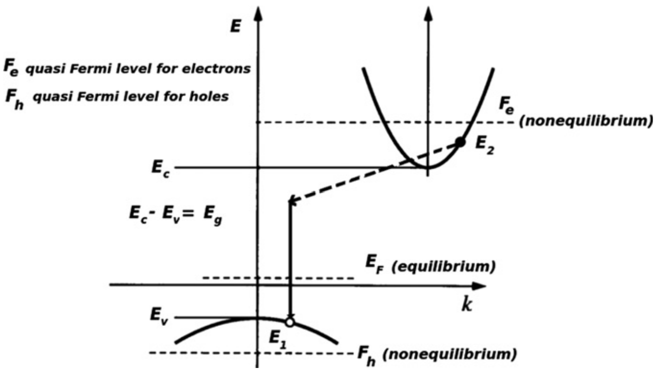

Fig. 3. Band structure of indirect bandgap semiconductor (for simplicity, we only consider two bands). We show a possible transition between CB and VB via a single-mode phonon and a single-mode photon.