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Disordered two-dimensional electron systems with chiral symmetry

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Dedicated to Costas Soukoulis on the occasion of his 60th birthday

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ABSTRACT

We review the results of our recent numerical investigations on the electronic properties of disordered two dimensional systems with chiral unitary, chiral orthogonal, and chiral symplectic symmetry. Of particular interest is the behavior of the density of states and the logarithmic scaling of the smallest Lyapunov exponents in the vicinity of the chiral quantum critical point in the band center at $E=0$. The observed peaks or depressions in the density of states, the distribution of the critical conductances, and the possible non-universality of the critical exponents for certain chiral unitary models are discussed.

1. Introduction

Two-dimensional disordered systems have been attracting special attention for many years because $d=2$ is the lower critical dimension of the metal-insulator transition (MIT) [1]. For lattice systems with orthogonal symmetry (random on-site disorder with time reversal symmetry) all electronic states are localized in the limit of infinite system size. However, for weak disorder and energies close to the band center, the localization length can become very large. On length scales smaller than the localization length, the wavefunctions exhibit self-similar (fractal) behavior [2]. The presence of spin dependent hopping changes the symmetry of the model to symplectic, and enables the system to undergo a metal–insulator transition at a certain value of the disorder strength [3–7]. The critical eigenstates at the MIT exhibit multifractal properties [8] and the localization length was reported to show a parity dependence [9]. A strong magnetic field turns the symmetry to unitary and induces critical states [10], i.e., singular energies where the localization length of the multifractal eigenstates [11,12] diverges, which are important for the explanation of dissipative transport [13,14] in the quantum Hall effect.

Two-dimensional (2D) models possessing an additional chiral symmetry exhibit various electronic properties not observed in the situations mentioned above. The chiral symmetry can be found in models defined on bi-partite lattices with non-diagonal disorder only [15,16]. Despite the disorder, the energy eigenvalues appear in pairs, E_n and $-E_n$ symmetrically around the band center $E=0$ (for the definition of chiral 2D models, see Section 2). Chiral symmetry implies unusual properties of the model in the vicinity of the band center $E=0$. For most chiral cases, the density of states and the localization length are diverging and the band center is a quantum critical point [17,18]. At zero temperatures, an infinite sample is metallic at $E=0$ but insulating for any non-zero energy. The appearance of the criticality at $E=0$ originates from the chiral symmetry. However, the existence of the critical point also depends on the boundary conditions. As listed in Table 1, the sample exhibits chiral symmetry only for special combinations of boundary conditions and parity. This boundary and parity dependence of the sample's length L_z and width L has no analogy in 'standard' disordered models.

The special symmetry of the energy spectra may be accompanied by a non-analytical behavior of the density of states (DOS) at the chiral critical point [18,23–25]. In 2D chiral unitary models defined on a bricklayer [26], which has the same topology as graphene's honeycomb lattice, the DOS exhibits a sharp drop near the band center going to zero at $E=0$ and depends on both disorder and system size [27]. Contrary to this behavior, the DOS of the chiral orthogonal system is finite at the band center and showing a narrow extra peak in the case of square lattice samples (see below).

Similar to other critical regimes, systems with chiral symmetry can be analyzed using the single parameter scaling theory [1]. However, the scaling parameter is not the ratio of the system size L to the correlation length $\xi(E)$, but the ratio of the logarithm of these parameters instead [28]

$$\chi = \frac{\ln L}{\ln \xi(E)/\xi_0}. \quad (1)$$

Also, the energy dependence of the correlation length is logarithmic [17,18,23]:

$$\ln(\xi(E)/\xi_0) \sim |\ln (E_0/|E|)|^{\kappa}, \quad (2)$$

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in contrast to the power-law scaling dependence $\xi(E) \sim |E - E_c|^{-\nu}$ observed in non-chiral disordered systems. Thus, models with chiral symmetry enable the detailed analysis of logarithmic scaling, discussed previously in Ref. [28].

Chirality also strongly affects the transport properties of the system. The non-Ohmic behavior of chiral systems with an odd number of open channels, where the mean conductance decreases much more slowly with the length of the system (see Fig. 1), has been predicted theoretically [22] and confirmed numerically in Refs. [26,29].

In this paper we review some results of our recent investigations on electric transport properties of two dimensional chiral systems. In the following section we briefly introduce the models studied. In Section 4, we summarize our findings for the Lyapunov exponents, the critical conductance and its probability distribution, and show recent calculations for the density of states. Of special interest is the scaling analysis of the diverging critical electronic states at $E=0$ [24,30,31].

Table 1
Two dimensional models with non-diagonal disorder possess the chiral symmetry only for special choices of the boundary conditions and the parity in the number of sites. For a given combination of boundary conditions, periodic (P) and Dirichlet (D), the chiral symmetry (Ch) and chiral symmetry with an extra eigenvalue at $E=0$ (Ch+) is observed [19–22].

$L_z \backslash L$	Odd			Even		
		D_x	P_x		D_x	P_x
Odd	D_z	**Ch+**		D_z	**Ch**	**Ch**
	P_z			P_z		
Even	D_z	**Ch**		D_z	**Ch**	**Ch**
	P_z	**Ch**		P_z	**Ch**	**Ch**

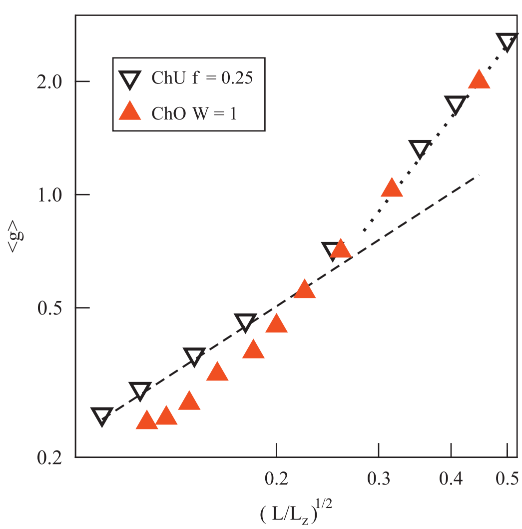

Fig. 1. The length dependence of the mean conductance $\langle g \rangle$ for quasi-one dimensional chiral systems of length L_z and fixed width $L=65a$. The energy is $E=0$. Due to the chiral symmetry of the model, the crossover from the Ohmic $1/L_z$-behavior (dotted line) to the $1/\sqrt{L_z}$ dependence (dashed line) is observed, in agreement with theoretical predictions [22]. Two models with chiral unitary (ChU: $f=0.25h/e$) and chiral orthogonal (ChO: $W/t_0=1$) symmetry, defined in Section 2, were considered.

2. Models

In the absence of diagonal disorder the single-band tight-binding Hamiltonian defined on the sites n of a two-dimensional bricklayer or square lattice with lattice constant a reads

$$\mathcal{H} = \sum_{\langle n \neq n' \rangle} t_{nn'} c_n^\dagger c_{n'}, \tag{3}$$

where the sum is over nearest neighbors only. The random disorder is incorporated in the hopping terms, which also determine the symmetry of the problem. Square lattice and bricklayer lattice differ only in the absence of every other vertical bond in the latter and so the coordination number is reduced to three (see Fig. 2).

2.1. Unitary symmetry

To describe a disordered chiral 2D system with broken time-reversal symmetry, the hopping terms in the (transversal) x direction are chosen to acquire complex phases and are defined as

$$t_x = t_0 e^{i\theta_{x,z;x\pm a,z}}, \tag{4}$$

where for a bricklayer lattice the phases $\theta_{x,z;x+a,z} = \theta_{x,z+2a;x+a,z+2a} - (2\pi e/h)\Phi_{x,z}$ are determined by the total flux threading the plaquette at (x,z)

$$\Phi_{x,z} = \frac{p}{q}\frac{h}{e} + \phi_{x,z}. \tag{5}$$

Here, p and q are mutual prime integers and the magnetic flux density perpendicular to the two-dimensional lattice $B = ph/(qe2a^2)$ is described by the number p/q of magnetic flux quanta h/e per plaquette $2a^2$. This differs from the random flux model studied previously [32], where the constant magnetic field part was absent. The random part is generated by the local fluxes $\phi_{x,z}$, which are uniformly distributed $-f/2 \le \phi_{x,z} \le f/2$ with zero mean. The disorder strength f can be varied within the interval from $f/(h/e)=0$ to $f/(h/e)=1$.

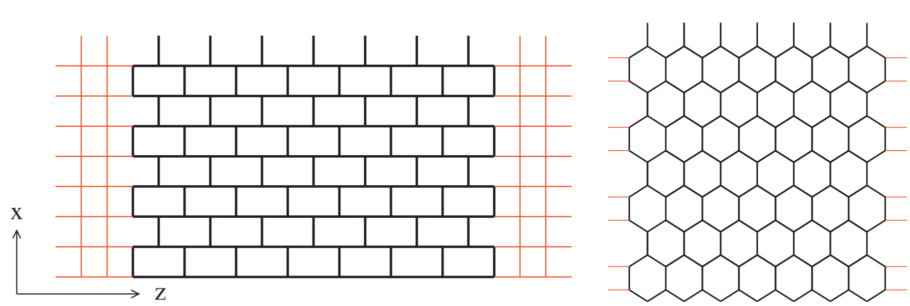

Fig. 2. The bricklayer lattice (left) shares the topology of the honeycomb lattice (right). The red lines display the attached perfect semi-infinite leads used in the calculation of the scaling variables z_i and the two-terminal conductance. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)