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Vector solitons in semiconductor quantum dots

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ABSTRACT

A theory of an optical vector soliton of self-induced transparency in an ensemble of semiconductor quantum dots is considered. By using the perturbative reduction method, the system of the Maxwell–Liouville equations is reduced to the two-component coupled nonlinear Schrödinger equations. It is shown that a distribution of transition dipole moments of the quantum dots and phase modulation changes significantly the pulse parameters. The shape of the optical two-component vector soliton with the sum and difference of the frequencies in the region of the carrier frequency is presented. The vector soliton can be reduced to the breather solution of self-induced transparency with a different profile. Explicit analytical expressions in the presence of single-excitonic and biexcitonic transitions for the optical vector soliton are obtained with realistic parameters which can be reached in current experiments.

1. Introduction

Semiconductor quantum dots (SQDs) as model systems for the consideration of coherent light–matter interaction have attracted much interest in the context of nano-optics and diverse applications. SQDs, also referred to as zero-dimensional systems, are nanostructures that allow confinement of charge carriers in all three spatial directions, which results in atomic-like discrete energy spectra and strongly enhanced carrier lifetimes. Such features make quantum dots similar to atoms in many respects (artificial atoms) [1]. Due to the large transition dipole moments of SQDs $\vec{\mu}$ reaching values on the order of 10^{-17} esu cm, the interaction between SQDs and optical pulses is strongly enhanced in comparison with atomic systems, making them especially attractive for investigations of nonlinear coherent optical phenomena. In addition, very long relaxation times at low temperatures in SQDs on the order of several tens of picoseconds [2,3] allow optical pulse propagation experiments to be performed with pulses of a few picoseconds.

The observation of optical coherence effects in ensembles of quantum dots is usually spoiled by the inhomogeneous line broadening due to dot size fluctuations, with typical broadenings comparable to the electronic level splitting. Quantum dots often have a base length in the range 50–400 Å. Size fluctuations in the quantum dot ensemble lead to an inhomogeneous single-exciton and biexciton level broadening, with a full width at half maximum of typically more than several tens of meV. Beside the frequency, the quantum dot size fluctuations influence also the transition dipole moments of the SQDs. Borri et al. [2] have reported measurements of optical Rabi oscillations in the excitonic ground-state transition of an inhomogeneously broadened InGaAs quantum dot ensemble. The effect of the biexcitonic resonance in the Rabi oscillations using different pulse durations have also been experimentally investigated. They found that a distribution with a 20% standard deviation of the transition dipole moments results in a strong damping of the oscillations versus pulse area. In the experiments reported in Ref. [2] it was also found that the period of the Rabi oscillations is changed. These results show quantitatively how uniformity in dot size is important for any application based on a coherent light-quantum dot ensemble interaction.

A nonlinear coherent interaction of an optical pulse with SQDs is governed by the Maxwell–Liouville equations. The large numerical values of the transition dipole moments and the inhomogeneous broadening of the spectral line do not change the Maxwell–Liouville equations for SQDs in comparison with atomic systems. But considering the distribution of the transition dipole moments of the SQDs, the polarization and Maxwell equation of the SQDs will differ in comparison with atomic systems because the polarization depends on the variable $\vec{\mu}$.

The existence of nonlinear solitary waves is one of the most interesting and important manifestations of nonlinearity in ensembles of SQDs. The determination of the mechanisms causing the formation of the optical nonlinear waves and the investigation of their properties are among the principal problems of the physics of SQDs.

A resonant optical nonlinear wave can be formed with the help of the resonance (McCall–Hahn) mechanism of the formation of nonlinear waves, i.e., from a nonlinear coherent interaction of an optical pulse with resonant ensembles of SQDs, when the conditions of self-induced transparency (SIT): $\omega T \gg 1$ and $T \ll T_{1,2}$ are fulfilled, where T and ω are the width and frequency of the pulse, and T_1 and T_2 the relaxation and dephasing time of the SQDs,

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respectively [3,4]. If the area of the pulse envelope as a measure of the light–matter interaction strength $\Theta > \pi$, a soliton is formed, and if $\Theta \ll 1$, a breather is generated.

Originally, SIT was investigated in atomic systems, but later, the search for solitary nonlinear waves was extended to ensembles of SQDs. Experimentally, self-induced transmission on a free exciton resonance in CdSe [5] and SIT in InGaAs quantum dot waveguide have been reported [6]. On the theoretical side, the effect of SIT for solitons and breathers in a sample of inhomogeneously broadened SQDs have been investigated numerically and analytically [3,7,4,8–10]. In these works, nonlinear waves are described by a single nonlinear Schrödinger (NLS) equation for a scalar one-component field. Such one-component nonlinear waves form when a single pulse propagates inside a medium containing SQDs in such a way that it maintains its state. When these conditions are not satisfied, one has to consider the interaction of the two field components at different frequencies or polarizations and solve simultaneously a system of two coupled NLS equations. A shape-preserving solution of these equations is a vector pulse because of its two-component nature.

In Ref. [11], from a nonlinear Klein–Gordon equation for a pulse envelope free of phase modulation, a vector soliton of SIT with the sum and difference of the frequencies (which were two or three order less in comparison with the carrier frequency) has been obtained. Under the condition of phase modulation, the physical situation will be different, and special considerations will be necessary. It is obvious that for an adequate description of SIT in SQDs, it is also necessary to take into account a distribution of transition dipole moments in the ensemble of quantum dots.

The main goals of this work are as follows: (i) the investigation of the processes of the formation of optical vector solitons with two different frequency components in the region of the carrier frequency. (ii) The determination of the explicit analytic expressions for the parameters of the two-component soliton with the sum and difference of the frequencies for the strength of the electric field of the wave. (iii) The analytical solution of the Maxwell–Liouville equations for the ensemble of SQDs in the presence of single-excitonic and biexcitonic transitions and the explanation how these equations are modified in the case of phase modulation and distribution of the dipole moments.

2. Basic equations

We consider the formation of optical nonlinear waves of SIT in an ensemble of SQDs for linearly polarized waves with width T and frequency $\omega \gg T^{-1}$ with an electric field strength $\vec{E}(z,t) = \vec{e}E(z,t)$ propagating along the positive z-axis, where $\vec{e}$ is a unit vector directed along the x-axis.

The pulse is tuned to transitions from the ground state $|1\rangle$ of the SQD to the states $|2\rangle$ and $|3\rangle$, with energies $\mathcal{E}_1 = 0$, $\mathcal{E}_2 = \hbar\omega_0 = \mathcal{E}_x + \delta_x/2$, and $\mathcal{E}_3 = \hbar\Omega_0 = 2\mathcal{E}_x + \delta_{xx}$, respectively. The quantities $\mathcal{E}_x = (\mathcal{E}_2 + \mathcal{E}'_2)/2$ and $\mathcal{E}_3$ are the energies of the single-excitonic and biexcitonic states, respectively. $\delta_x = \mathcal{E}_2 - \mathcal{E}'_2$ and δ_{xx} are the energies of the exciton fine structure splitting and biexcitonic binding energy (negative if bound), respectively (Fig. 1); $\hbar$ is Planck's constant. In order that $\delta_x/2 \ll \hbar\omega_0$ and $\delta_{xx} \ll \hbar(\Omega_0 - \omega_0)$, the $|1\rangle$ to $|2\rangle$ transition and the $|2\rangle$ to $|3\rangle$ transition are very close to each other and to the pulse frequency ω. The energetic spectrum of the quantum dots can be considered as a quasi-equidistant three-level system in a cascade configuration under off-resonant excitation $\Omega_0 - \omega_0 - \omega \neq 0$ and $\omega_0 - \omega \neq 0$. We assume that the detunings from the resonance $\Omega_0 - \omega_0 - \omega$ and $\omega_0 - \omega$ lie within the bandwidth of the pulse [7].

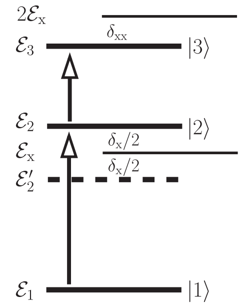

Fig. 1. Schematic of the SQD energetic levels.

The Hamiltonian of the system is given by

$$H = H_0 + V, \tag{1}$$

where $H_0 = \hbar\omega_0|2\rangle\langle 2| + \hbar\Omega_0|3\rangle\langle 3|$ describes the kinetics of the single-excitonic and biexcitonic states and $V = -\vec{P}\cdot\vec{E}$ is the Hamiltonian of the light-quantum dot interaction. The vector of polarization

$$\vec{P}(z,t) = \frac{n_0}{2}\sum_{l=\pm 1}\vec{e}_p Z_l\langle \mu p_l\rangle, \tag{2}$$

where n_0 is the uniform dot density, $\vec{e}_p$ is the unit vector of polarization, $Z_l = \exp[il(kz - \omega t)]$. $p_l = p^*_{-l}$ is the slowly changing complex amplitude of the polarization. In the case of a cascade configuration $\mu_{13} = 0$ and $p_1 = \mu_{12}\rho_{21} + \mu_{23}\rho_{32}$, where $\mu_{12} = \vec{\mu}_{12}\cdot\vec{e}$, $\mu_{13} = \vec{\mu}_{13}\cdot\vec{e}$, $\mu_{23} = \vec{\mu}_{23}\cdot\vec{e}$. The quantities $\vec{\mu}_{12}$ and $\vec{\mu}_{23}$ are the dipole moments for the corresponding transitions which we assume to be parallel to each other. The quantities ρ_{nm} are the elements of the density matrix ρ which are determined by the Liouville equation [13]

$$i\hbar\frac{\partial\rho_{nm}}{\partial t} = \sum_k(\langle n|H|k\rangle\rho_{km} - \rho_{nk}\langle k|H|m\rangle),$$

where $n,m,k = 1,2,3$. Substituting in this equation the expression for the Hamiltonian (1), we obtain a system of equations for the elements of the density matrix for the quantum dot ensemble

$$i\hbar\frac{\partial\rho_{11}}{\partial t} = (-\mu_{12}\rho_{21} + \mu^*_{12}\rho_{12})E,$$

$$i\hbar\frac{\partial\rho_{22}}{\partial t} = (\mu_{12}\rho_{21} - \mu_{23}\rho_{32} - \mu^*_{12}\rho_{12} + \mu^*_{23}\rho_{23})E,$$

$$i\hbar\frac{\partial\rho_{33}}{\partial t} = (-\mu^*_{23}\rho_{23} + \mu_{23}\rho_{32})E,$$

$$i\hbar\frac{\partial\rho_{21}}{\partial t} = \hbar\omega_0\rho_{21} - \mu^*_{12}E(\rho_{11} - \rho_{22}) - \mu_{23}E\rho_{31},$$

$$i\hbar\frac{\partial\rho_{32}}{\partial t} = \hbar(\Omega_0 - \omega_0)\rho_{32} + \mu_{12}E\rho_{31} - \mu^*_{23}E(\rho_{22} - \rho_{33}),$$

$$i\hbar\frac{\partial\rho_{31}}{\partial t} = \hbar\Omega_0\rho_{31} - \mu^*_{23}E\rho_{21} + \mu^*_{12}E\rho_{32}. \tag{3}$$

When the transitions between energetic states of the quantum dots correspond to a $\Delta m = 0$ transition, we may take μ_{21} and μ_{23} to be real vectors, $\mu_{21} = \mu^*_{21}$, $\mu_{23} = \mu^*_{23}$; such transitions might be induced by linearly polarized light which is investigated in detail. In addition, for simplicity, we assume that $\mu = \mu_{12} = \mu_{23}$ and under this condition

$$\langle \mu p_l\rangle = \int\int g(\Delta)h(\mu_0 - \mu)\mu p_l(\Delta,\mu,z,t)\,d\mu\,d\Delta, \tag{4}$$

where $g(\Delta)$ is the inhomogeneous broadening lineshape function, $\Delta = \omega_0 - \omega$, and $h(\mu_0 - \mu)$ is the distribution function of transition dipole moments of SQDs. For this function the normalization