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Effect of the internal rib structure of the inclusions on the two-dimensional phononic crystal composed of periodic slotted tubes in air

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ABSTRACT

Using a finite element method based on the Bloch theorem, a new phononic crystal composed of periodic slotted tubes with internal rib structure in air is investigated. Two parallel plates with slit are introduced into the inclusion as the internal rib structure and its effect on band gaps is studied. The band structure and acoustic modes of the PC are calculated. Results show that the starting frequency of the first band gap is rather lower than that of slotted tubes without rib structure. The internal rib structure plays an important role in both the lower and upper edges of the first band gap. Some rib structural parameters are also studied for their effects on the first band gap. Results show that the first gap can be modulated widely by these parameters and the Helmholtz resonator theory can be used to explain the relationship between the band gap and the parameters.

1. Introduction

In the last two decades, the propagation of elastic waves in periodic composite media, known as phononic crystals (PCs), has received growing interest because of their potential applications and unique physical properties [1–4]. Phononic crystals are artificial mediums made of at least two materials with different elastic constants. One of them is periodically embedded in a host matrix composed of the other material [5,6]. In such mediums, there exist some frequency ranges in which the propagation of elastic waves is prohibited, known as the phononic band gaps (PBGs) [7,8]. Since the PBGs can be used to realize some fundamental functionalities such as sound shields, filtering, guiding [9–13], as well as for pure physics studies concerned with the Anderson localization of sound and the negative refraction [14,15], many studies on the research of PBGs were carried out. Kushwaha and Djafari-Rouhani [16] computed extensive band structures for two-dimensional periodic arrays of air cylinders in water and obtained complete, multiple and wide band gaps. Lambin and Khelif [17] found that hollow cylinders made of an elastically-soft polymer containing air inside and arranged on a square lattice in water can also produce large acoustic band gaps. Pennec et al. [18] obtained full band gaps in the PCs composed of hollow steel cylinders in water, and discussed the transmission characteristics along the principal high-symmetry direction. Meanwhile, the effects of structure parameters and the physics parameters of the PCs such as the lattice constant and density ratio on the band gaps were also presented [19–22].

One of the motives for studying the PCs is to pursuit the wider band gaps with lower starting frequency for the applications in the audible range. Researchers have tried different methods to achieve this goal. Caballero et al. [23] studied the two-dimensional square and triangular lattices of rigid cylinders in air and found that the absolute acoustic band gaps can be increased by reducing the structure symmetry. Kushwaha and Djafari-Rouhani [24] designed a tandem structure composed of cylinder inclusions with different radii, and the results showed that this new PCs can yield a wide band gap of thousands Hz. Li et al. [25] obtained wide band gaps in 2D solid/air PCs by rotating the square rod inclusions. Lai et al. [26] found that the phononic band gaps can be enlarged by altering the microstructure of the square lattice of steel cylinders in air background. Although these attempts can widen the band gaps, the low starting frequencies of PBGs are difficult to obtain as the Bragg scattering mechanism requires that the dimension of the PCs is in the same order as the wavelength of the acoustic waves, which limited the application of PCs in low-frequency range.

To overcome the limitation of the Bragg band gaps, Liu et al. [27] studied the acoustic waves propagating in three-component PC composed of periodic coated lead balls immersed in an epoxy matrix, and obtained band gaps two orders of magnitude smaller than that of the Bragg gap. Based on this result, a new gap mechanism, the locally resonant mechanism (LR), was proposed. Wang et al. [28] studied the propagation of elastic waves in two-dimensional LRPCs composed of periodic soft rubber/epoxy composites and found that the 2D system can also produce locally

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resonant band gaps. Sainidou et al. [29] demonstrated that the three- and two-dimensional phononic crystals composed of hollow spheres or cylinders exhibit localized resonance states. This result implied that the LR band gaps can be obtained by not only the coated structures but also some other structures with resonant characteristics. Recently, Cui et al. [30] investigated a band gap structure composed of a square array of parallel steel tubes with narrow slits both theoretically and experimentally. They found that the Bragg gaps and locally resonant gaps simultaneously existed in this PC. The effects of some structural parameters of the tube inclusion such as the slit width and the slit number on the band gaps were also studied. Results showed that these parameters can modulate the locally resonant gaps in some extent. By arranging inclusions with different slit widths in the PCs, the Bragg gap and the locally resonant gap can be combined, resulting in a large band gap with low starting frequency. As we know, besides the slit parameters, the internal structure of the slotted tube can also influence the resonant characteristics of the inclusion. So in our opinion, the band gap of the PCs can be further modulated by the internal structure of the tubes. It is very interesting and valuable that the band gaps varied while the external dimensions of the PC structure keep unchanged. However, the research on this field has not yet been reported.

In this paper, a new band gap structure composed of periodic slotted steel tubes with internal rib structure in air is considered. Two parallel plates with slit are introduced into the tubes to form the internal rib structure of the inclusion. The band structures are calculated theoretically using a finite element method based on the Bloch theorem. To prove the correctness of the calculation results, the transmission spectrum of a finite structure is also calculated by the FEM similar with the method in Ref. [30]. The acoustic modes of the bands are calculated to better understand the role that the internal rib structure played on the band gaps of the slotted tube PCs. Then some numerical results are used to show how the locally resonant band gap depends on the parameters of the internal rib structure such as the thickness of the inner plates and the distance between the two plates. Finally, some conclusions are given.

2. Model and methods of calculation

The PC structure considered here is composed of periodic steel slotted tubes with internal rib structure in air matrix with a square lattice, Fig. 1(a) and (b) shows the unit cell of the PC. The original inclusions of the PC are the slotted tubes whose inner and outer diameters are r_i and r_o, respectively. By setting two parallel plates with slit inside the tube, the internal rib structure is formed and the original resonator cavity is divided into two cavities and three channels. The geometrical parameters of the internal rib structure are defined as follows: the thickness of the plate and the distance of the two parallel plates are t and d respectively; and the width of the slit of the plate is denoted by w. The complete structure is an infinite repetitive arrangement of the unit cells along the x- and the y- directions. The lattice spacing of the square lattice and the slit width is denoted by a and s, respectively.

In order to investigate the acoustic forbidden bands and resonant modes in the proposed PC structure, a series of calculations of the dispersion relationships are conducted with FEM [31,32]. Since the infinite system is periodic along the x- and y- directions simultaneously, only the unit cell shown in Fig. 1(b) needs to be considered. In the calculations, the unit cell is divided into two domains, the air domain and the solid domain. For the air domain, the governing equation for the acoustic waves is given by the frequency-domain Helmholtz equation

$$\nabla\left(-\frac{1}{\rho_0}\nabla p\right)=\frac{\omega^2 p}{\rho_0 c_s^2}, \tag{1}$$

where p is the acoustic pressure, ρ_0 is the density, ω is the angular frequency and c_s is the speed of sound.

For the solid domain, as the acoustic impedance of air is much smaller than that of steel, the total longitudinal waves propagating in air will be almost reflected by the steel tubes. The wave propagation in such PC is predominant in the air domain. So, the transverse waves in steel tubes can be ignored. In order to achieve this approximation, we can consider the steel tubes as fluid with very high stiffness and specific mass. With the Bloch theorem, periodic boundary conditions are applied at the boundaries between the unit cell and its four adjacent cells

$$p(\mathbf{r}+\mathbf{a})=p(\mathbf{r})e^{i\mathbf{Ka}}, \tag{2}$$

where $\mathbf{r}$ is located at the boundary nodes and $\mathbf{a}$ is the basis vector of the periodic structure.

The parameter $\mathbf{K}$ is defined as a 2D Bloch wave vector to describe the phase relationship of a plane wave between the boundaries of the unit cells. With a given value of $\mathbf{K}$, a group of eigenvalues and eigenmodes can be obtained by solving the eigenvalue problem generated by FEM algorithm. By changing the value of $\mathbf{K}$ along the boundary of the first Brillouin zone and

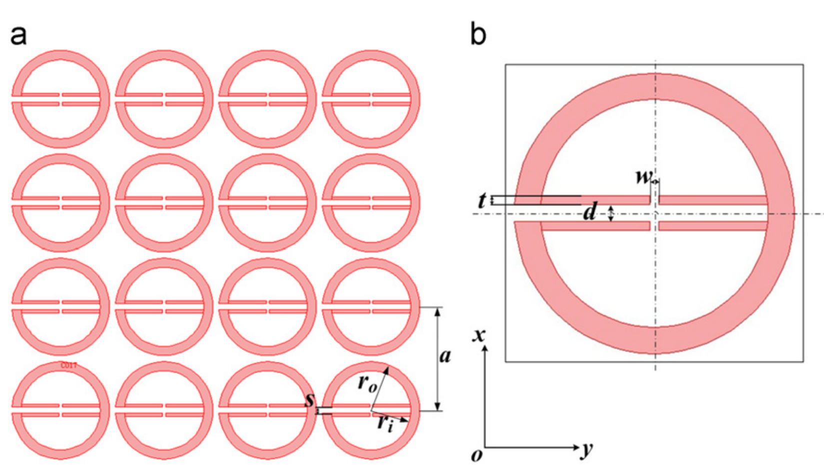

Fig. 1. (a) Schematics of the gap structure composed of a periodic square array of slotted tubes with internal rib structure in air matrix. The lattice constant is a; r_o and r_i are the outer and inner radius of the slotted tube; the width of the slit on the tube is s. (b) Schematics of the unit cell of the gap structure, in which t is the thickness of the plates inside the tube, d is the distance of the two plates and w is the width of slit of the plates.