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Data-driven techniques to estimate parameters in a rate-dependent ferromagnetic hysteresis model

Zhengzheng Hu [a], Ralph C. Smith [a,*], Jon M. Ernstberger [b]

[a] Department of Mathematics and Center for Research in Scientific Computation, North Carolina State University, Raleigh, NC 27695, USA
[b] Department of Mathematics, 601 Broad Street, LaGrange College, LaGrange, GA 30240, USA

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ABSTRACT

The quantification of rate-dependent ferromagnetic hysteresis is important in a range of applications including high speed milling using Terfenol-D actuators. There exist a variety of frameworks for characterizing rate-dependent hysteresis including the magnetic model in Ref. [2], the homogenized energy framework, Preisach formulations that accommodate after-effects, and Prandtl–Ishlinskii models. A critical issue when using any of these models to characterize physical devices concerns the efficient estimation of model parameters through least squares data fits. A crux of this issue is the determination of initial parameter estimates based on easily measured attributes of the data. In this paper, we present data-driven techniques to efficiently and robustly estimate parameters in the homogenized energy model. This framework was chosen due to its physical basis and its applicability to ferroelectric, ferromagnetic and ferroelastic materials.

1. Introduction

The quantification of rate-dependent effects, including accommodation or reptation, magnetic after-effects, and eddy current effects, is crucial when employing magnetic transducers in high drive regimes. There exist a number of modeling frameworks that incorporate these effects, for materials acting in hysteretic regimes, including the magnetic models of Ref. [2], homogenized energy models (HEM) [9,10], extended Preisach models [5,7], and rate-dependent Prandtl–Ishlinskii models [1]. A critical issue when employing any of these frameworks concerns the robust determination of model parameters based on least squares fits to data. In this paper, we present a data-driven technique to obtain initial parameter estimates for the homogenized energy model for ferromagnetic hysteresis. We employ this framework due to its energy basis at mesoscales, its ease of implementation and relative accuracy for a range of operating conditions, the degree to which model parameters can be correlated with physical properties of measured data, and its ubiquity for ferromagnetic, ferroelectric (e.g., PZT), and ferroelastic (e.g., SMA) materials.

We focus on rate-dependent effects and creep behavior associated with magnetic after-effects [4] since, for the transducer designs and data under consideration, eddy current losses have been minimized. The extension of these models and data-driven parameter estimation techniques to incorporate eddy currents is under current investigation.

A short description of the homogenized energy model is provided in Section 2. In Section 3, we detail the data-driven techniques to efficiently and robustly estimate parameters in the homogenized energy model. In Section 4, we compare the model fits to experimental data for both steel and nickel rods.

2. Homogenized energy model for magnetic hysteresis

As detailed in Refs. [9,10], the homogenized energy model for ferromagnetic materials is constructed in two steps. In the first, a Gibbs energy is balanced with a relative thermal energy to construct a local average magnetization relation. The effects of material and field nonhomogeneities are subsequently incorporated by assuming that local coercive and interaction fields are manifestations of underlying densities rather than constant parameters. Stochastic homogenization in this manner yields a macroscopic model that is accurate for a variety of operating regimes and is efficient to construct and implement.

For 180° moment switching, we employ the Gibbs energy

$$G(H,M) = \psi(M) - HM, \tag{1}$$

where H and M respectively denote the field and magnetization, and the Helmholtz energy is

$$\psi(M) = \begin{cases} \frac{1}{2}\eta(M+M_R)^2, & M \le -M_I \\ \frac{1}{2}\eta(M-M_R)^2, & M \ge M_I \\ \frac{1}{2}\eta(M_I-M_R)\left(\frac{M^2}{M^I}-M_R\right), & |M| < M_I. \end{cases}$$

* Corresponding author. Tel.: +1 919 515 7552; fax: +1 919 515 1636.
E-mail addresses: zhu4@ncsu.edu (Z. Hu), rsmith@ncsu.edu (R.C. Smith), jernstberger@lagrange.edu (J.M. Ernstberger).

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Here $\eta = dH/dM$ after switching and the remanent magnetization M_R are two of the parameters to be identified.

As detailed in Refs. [9,10], thermal relaxation is accommodated by balancing the Gibbs energy and relative thermal energy kT/V using the Boltzmann relation:

$$\mu(G) = Ce^{-GV/kT}, \tag{2}$$

where k is the Boltzmann constant and T is the temperature in degree Kelvin. The reference volume V reflects the definition of magnetization as magnetic moments per unit volume and yields a relative thermal energy based on the Gibbs energy density.

Approximation of the relations

$$\langle M_+ \rangle = c\int_{M_I}^{\infty} Me^{-G(H,M)V/kT}\, dM \tag{3}$$

and

$$\langle M_- \rangle = c\int_{-\infty}^{-M_I} Me^{-G(H,M)V/kT}\, dM \tag{4}$$

yields the relation

$$\overline{M} = 2M_R x_+ + \frac{H+H_I}{\eta} - M_R \tag{5}$$

for the local average magnetization. Here x_+ denotes the fraction of positively oriented moments which evolve via the differential equation

$$\dot{x}_+ = -(p_{+-}+p_{-+})x_+ + p_{-+}, \tag{6}$$

involving the likelihoods of moments switching from negative to positive (p_{+-}) and conversely (p_{-+}). As detailed in Ref. [3], we employed the likelihood relations

$$p_{+-} = \frac{\gamma_1}{\text{erfcx}(H_+)}, \quad p_{-+} = \frac{\gamma_1}{\text{erfcx}(H_-)}, \tag{7}$$

where $\text{erfcx}(x) = e^{x^2}(2/\sqrt{\pi})\int_x^\infty e^{-t^2}\, dt$ (the scaled complementary error function), $H_+ = -\gamma_2(H_c+(H+H_I))$, $H_- = -\gamma_2(H_c-(H+H_I))$, $\gamma_1 = (1/\tau)\sqrt{(2V\eta/\pi kT)}$ and $\gamma_2 = \sqrt{(V/2kT\eta)}$. Here τ is the relaxation time, or the reciprocal of the frequency at which moments attempt to switch.

To incorporate the effects of polycrystallinity, material nonhomogeneities, and variable interaction fields, we subsequently assume that local coercive and interaction fields are manifestations of underlying densities rather than constant coefficients. This yields the macroscopic magnetization relation:

$$M(H(t)) = \int_0^\infty \int_{-\infty}^{\infty} \overline{M}(H(t)+H_I; H_c) \times \nu_c(H_c)\nu_I(H_I)\, dH_I\, dH_c, \tag{8}$$

where $\nu_c(H_c)$ and $\nu_I(H_I)$ are densities associated with the coercive and interaction fields. Various quadrature techniques, including midpoint or Gaussian rules, can be used to approximate two integrals in Eq. (8).

Following Ref. [6], we employ expansions

$$\nu_c(H_c) = c_1 \sum_{k=k_0,m=1}^{k_1,M_\alpha} \alpha_{k,m}\phi_{k,m}(H_c), \tag{9}$$

$$\nu_I(H_I) = c_2 \sum_{k=k_0}^{k_1} \beta_k \psi_k(H_I), \tag{10}$$

where the basis functions $\phi_{k,m}(H_c)$ and $\psi_k(H_I)$ are lognormal and normal distributions and the coefficients $\alpha_{k,m}$ and β_k are determined through a least squares fit to data. The constants $c_1 = (\sum_{k=k_0,m=1}^{k_1,M_\alpha} \alpha_{k,m})^{-1}$ and $c_2 = (\sum_{k=k_0}^{k_1} \beta_k)^{-1}$ ensure integration to unity. Examples of the two densities are shown in Fig. 1.

The coercive field basis functions are taken to be lognormal distributions:

$$\phi_{k,m}(H_c) = \frac{1}{\sigma_{c_k}\sqrt{2\pi}H_c}\exp\left(-\frac{(\ln H_c - \ln \overline{H}_{c_m})^2}{2\sigma_{c_k}^2}\right), \tag{11}$$

whose corresponding normal distribution has mean $\ln \overline{H}_{c_m}$ and standard deviation $\sigma_{c_k} = 2^k\sigma_c$. As detailed in Section 3, one of the $\overline{H}_{c_m}$ (e.g., $m=2$) can be obtained directly from the coercive field of the data. Once $\overline{H}_{c_2}$ is determined, $\overline{H}_{c_1}$ and $\overline{H}_{c_3}$ can be determined by, for example, letting $\overline{H}_{c_1} = 1.5\overline{H}_{c_2}$ and $\overline{H}_{c_3} = 0.5\overline{H}_{c_2}$.

The interaction field basis functions are taken to be normal distributions:

$$\psi_k(H_I) = \frac{1}{\sigma_{I_k}\sqrt{2\pi}}\exp\left(-\frac{H_I^2}{2\sigma_{I_k}^2}\right), \tag{12}$$

with mean 0 and standard deviation $\sigma_{I_k} = 2^k\sigma_I$ for σ_I where σ_I can be obtained from the data.

3. Data-driven parameter estimation

3.1. Optimization method

The parameters to be estimated are

$$q = \{\eta, M_R, \overline{H}_{c_2}, \sigma_I, \sigma_c, \gamma_1, \gamma_2, \alpha_{k,m}, \beta_k\}. \tag{13}$$

To ensure the accuracy of nested, biased minor loops, we take $k = -2,-1,0,1$, and $m = 1,2,3$. For simpler operating regimes, fewer values can be used.

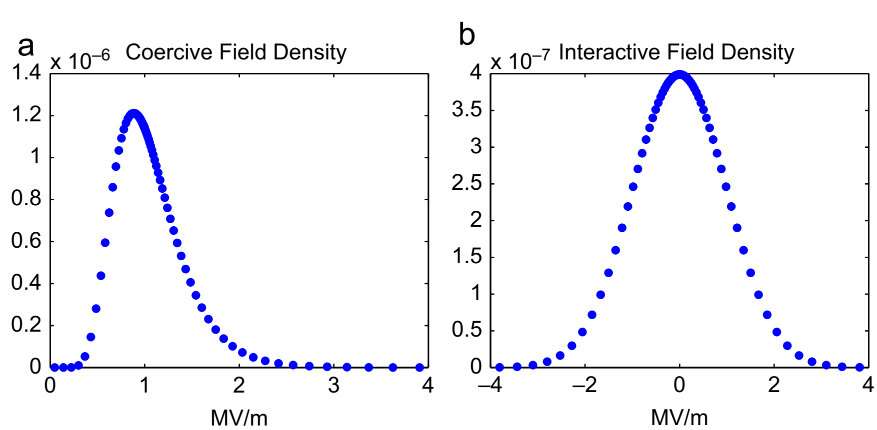

Fig. 1. (a) Sampled lognormal density ν_c with 80 points. (b) Sampled normal density ν_I with 80 points.