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Dwell time in one-dimensional graphene asymmetrical barrier

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ABSTRACT

The authors have investigated theoretically the dwell time of Dirac fermions tunneling through electrostatic square barrier in monolayer graphene, including asymmetrical and symmetrical potential barriers. It is found that the incident angle determines the critical incident energy. When the incident energy is larger than the critical incident energy, the dwell time saturate with the increase of the barrier thickness. But when the incident energy is smaller than the critical incident energy, the dwell time oscillates with the increase of the barrier thickness. The behaviors of oscillation and saturation of the dwell time are related with the transmission probability. These results may be helpful for the basic physics and potential application of graphene based electronic devices.

1. Introduction

The question of how long it takes a particle to tunnel through a potential barrier is one that has occupied physicists since the early days of quantum mechanics [1,2]. The tunneling time of a particle through a barrier or well is also a very important parameter in many electronic devices, for example metal–insulator–metal thin film sandwiches, and thence has received much attention in the past several years. There are two important qualities called group delay time τ_g and dwell time τ_d, respectively. The group delay time τ_g can be expressed in terms of the derivative of the phase shift with respect to energy [3], and Büttiker and Landauer thought the group delay was not a physically meaningful quantity with which to characterize tunneling dynamics [4]. The dwell time was first introduced by Smith in a potential scattering context, which was defined as the difference between the time spent by a particle in the region of the scattering potential and the time spent in the same region in the absence of the scattering potential [5]. Hartman found that the group delay time τ_g for a particle tunneling through a rectangular barrier is independent of the barrier thickness if the barrier is opaque, which is often referred to as the Hartman effect [4]. Some experiments [6–10] have verified this phenomenon and gave a possible reason that the wavepacket is merely reshaped in the tunneling process. And Winful considers the delays are not propagation delays but the momentary capture and release of a tunneling particle [11–14].

Graphene is a one-atom-thick planar sheet of carbon atoms that are densely packed in a honeycomb crystal lattice [15]. The fabrication of graphene overthrows the prediction that strictly two-dimensional crystal cannot exist in finite temperature because of thermal disturbance [16]. Graphene has many novel properties and potential applications, for example, the prediction and observation of half-integer quantum hall effect [17], finite conductivity at zero charge carrier concentration [18], perfect quantum tunneling effect [19], and ultrahigh carrier mobility [20]. And the quasiparticles called massless Dirac fermions can be viewed as electrons that lose their rest mass, which are described by the massless Dirac equation rather than the Schrödinger equation in conventional semiconductor structures. The Hartman effect in graphene has been investigated involving in quantum tunneling through single and double symmetric square barriers [21,22] in a single layer graphene, and the authors find that there is Hartman effect in graphene. But Dragoman et al. shows that there is no Hartman effect due to Klein paradox for electrons traversing electrostatic barriers in graphene [23]. They found the electrostatic barriers cannot confine charge carriers because of no bandgap in graphene. Their essential difference from the previous results [21,22] is that they found the imaginary wavenumbers do not exist in graphene. On the other hand, the dwell time in graphene-based magnetic barrier nanostructures including the square magnetic barrier and the δ-function magnetic barrier has been investigated [24], and the authors found that both the dwell time and the transmission probability showed remarkable anisotropy varying in different magnetically modulated configurations.

In this paper, we study theoretically the dwell time in a single layer graphene, where the potential barriers are asymmetric. We calculate the dwell time τ_d and the calculation results show that there is obvious saturation of the dwell time in graphene asymmetric barriers, and when the asymmetric barriers tend to symmetric barriers there is still saturation effect. What determines the existence

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of the saturation of dwell time are the incident energy and angle. This paper is organized as follows. We give the theoretical model and the calculation method in Section 2. And in Section 3 we show the numerical results and discussions. Finally, we give the conclusions in Section 4.

2. The formula and model of the dwell time

Now we consider a monolayer graphene sheet in the (x, y) coordinate plane, where a gate electrode is patterned to implement a rectangular potential barrier with the height V_1 and add a bias between the barrier to make the right potential of the barrier V_2. The potential barrier is shown schematically in Fig. 1, where the thickness of the barrier is D along the y direction between the $x=0$ and $x=D$ planes. In our study, the Dirac fermion with energy E and momentum $\hbar k$ is incident from the left side with incident angle θ on the potential barrier. The wave function of a low energy excited electron, which locates in the vicinity of the Dirac point satisfies the two-dimensional time-independent Dirac-like equation. The Hamiltonian of the massless Dirac fermion in the low energy region reads as: $\hat{H}=-i\hbar v_F\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}$ [20], where $\hbar$ is the reduced planck constant, $v_F=10^6$ m/s is the Fermi velocity, k is the wave vector, and $\boldsymbol{\sigma}=(\sigma_x,\sigma_y)$ are the Pauli spin matrices. The two-component electron wavefunction $\psi(x,y)$ is given by the two-dimensional massless Dirac equation, and the two-component wavefunctions of the left, middle and right region $\Psi_{\rm I}(x,y)$, $\Psi_{\rm II}(x,y)$ and $\Psi_{\rm III}(x,y)$ are described by

$$\begin{cases}\Psi_{\rm I}(x,y)=\dfrac{1}{\sqrt{2}}\begin{bmatrix}1\\ \frac{k_{x1}+ik_y}{E}\cdot\hbar v_F\end{bmatrix}e^{ik_{x1}x+ik_yy}+\dfrac{r}{\sqrt{2}}\begin{bmatrix}1\\ \frac{-k_{x1}+ik_y}{E}\cdot\hbar v_F\end{bmatrix}e^{-ik_{x1}x+ik_yy},\\ \Psi_{\rm II}(x,y)=\dfrac{a}{\sqrt{2}}\begin{bmatrix}1\\ \frac{k_{x2}+ik_y}{E-V_1}\cdot\hbar v_F\end{bmatrix}e^{ik_{x2}x+ik_yy}+\dfrac{b}{\sqrt{2}}\begin{bmatrix}1\\ \frac{-k_{x2}+ik_y}{E-V_1}\cdot\hbar v_F\end{bmatrix}e^{-ik_{x2}x+ik_yy},\\ \Psi_{\rm III}(x,y)=\dfrac{t}{\sqrt{2}}\begin{bmatrix}1\\ \frac{k_{x3}+ik_y}{E-V_2}\cdot\hbar v_F\end{bmatrix}e^{ik_{x3}x+ik_yy},\end{cases}\tag{1}$$

where $k_{x1}=(E/\hbar v_F)\cos(\theta)$ and $k_y=(E/\hbar v_F)\sin(\theta)$ are the perpendicular and the parallel wave vector components of the left side, θ is the incident angle,

$$k_{x2}=\mathrm{Sgn}(E-V_1)\sqrt{\left(\frac{E-V_1}{\hbar v_F}\right)^2-k_y^2}$$

and

$$k_{x3}=\mathrm{Sgn}(E-V_2)\sqrt{\left(\frac{E-V_2}{\hbar v_F}\right)^2-k_y^2}$$

are the perpendicular wave vector components of the middle and right side, respectively. Here we only focus on the situation where the energy of the incident electron E is lower than the barrier height V_1.

In one-dimensional quantum system it has been proved that the dwell time is equal to the group delay time minus a self-interference delay [12]. The relationship between the group delay time and the dwell time in graphene through a symmetric barrier was studied by Wu et al. [21]. They found that due to the linear dispersion relation, the group delay time equals the dwell time [21]. Based on the approach, we study the relationship between the group delay time and the dwell time through an asymmetric barrier in graphene. The Hamiltonian is $\hat{H}=v_F\vec{\sigma}\cdot\hat{p}$, where $v_F\approx 10^6$ m/s is the Fermi velocity, and $\vec{\sigma}$ is Pauli's matrices. We make $t=|t|e^{i\phi_t}$ and $r=|r|e^{i\phi_r}$, where ϕ_t and ϕ_r are the transmission and reflection phase angles. The definitions of the group delay time and the dwell time are $\tau_g=|t|^2\cdot\hbar d\phi_o/dE+|r|^2\cdot\hbar d\phi_r/dE$, and $\tau_d=\int_0^D|\psi(x)|^2\,dx/j_{in}$, where $\phi_o=\phi_t+k_{x3}D$, and $j_{in}=v_F\cos(\theta)$ is the flux of incident particles [5,21]. In the presence of electrostatic potential V, the wavefunction equation reads as

$$(\hat{H}-E)\psi=(\overrightarrow{T}+V-E)\psi=0,\tag{2}$$

where $\overrightarrow{T}=-i\hbar v_F\vec{\sigma}\cdot\overrightarrow{\nabla}$. We can obtain $\psi^+\psi=-i\hbar v_F\ \nabla\cdot(\psi^+\vec{\sigma}\partial\psi/\partial E)$. So according to the definition of the dwell time we obtain

$$\tau_d=\frac{\int_0^D|\psi|^2\,dx}{v_F\cos\theta}=\frac{-i\hbar}{\cos\theta}\cdot\left[\left(\psi^+\vec{\sigma}_x\frac{\partial}{\partial E}\psi\right)_{x=D}-\left(\psi^+\vec{\sigma}_x\frac{\partial}{\partial E}\psi\right)_{x=0}\right].$$

The dwell time is as follows:

$$\begin{aligned}\tau_d&=\hbar\left[|t|\frac{d|t|}{dk}+|t|^2\left(\frac{d\phi_0}{dE}+\sin\phi\cdot y\right)\right]\\&\quad-\hbar\left[\sin\phi\cdot y-|r|\frac{d|r|}{dE}-|r|^2\left(\frac{d\phi_r}{dE}+\sin\phi\cdot y\right)\right]\\&=\hbar\left(|t|^2\frac{d\phi_0}{dE}+|r|^2\frac{d\phi_r}{dE}\right).\end{aligned}\tag{3}$$

We find that the dwell time is also identical to the group delay time in graphene asymmetrical barrier, which means the self-interference delay is zero. In the following, we calculate the dwell time through the potential barrier using the following general formula [5,21]:

$$\tau_d=\frac{\int_0^D|\psi|^2dx}{v_F\cos\theta}.\tag{4}$$

According to the boundary conditions, the wavefunctions at $x=0$ and D should be continuous. We can obtain all of the above coefficients of the functions in Eq. (1). In order to obtain the dwell time we only concern the wavefunction in the barrier region, so we just need to obtain a and b in Eq. (1) as follows:

$$\begin{cases}a=\dfrac{2k_{x1}(E-V_1)*C_2}{C_1*C_2+C_3*C_4*e^{2ik_{x2}D}},\\ b=\dfrac{2k_{x1}(E-V_1)*C_4}{C_1*C_2*e^{-2ik_{x2}D}+C_3*C_4},\end{cases}\tag{5}$$

where

$$\begin{aligned}C_1&=(E-V_1)(k_{x1}-ik_y)+E(k_{x2}+ik_y),\\C_2&=(E-V_2)(k_{x2}-ik_y)+(E-V_1)(k_{x3}+ik_y),\\C_3&=(E-V_1)(k_{x1}-ik_y)-E(k_{x2}-ik_y),\\C_4&=(E-V_2)(k_{x2}+ik_y)-(E-V_1)(k_{x3}+ik_y).\end{aligned}\tag{6}$$

And we can obtain $|\psi|^2=\psi^+\psi$ in the barrier region as follows:

$$\psi^+\psi=\begin{cases}|a|^2+|b|^2+\dfrac{a^*b}{2}\left[1-\dfrac{\hbar^2v_F^2(k_{x2}-ik_y)^2}{(E-V_1)^2}\right]e^{-2ik_{x2}x}\\ \quad+\dfrac{ab^*}{2}\left[1-\dfrac{\hbar^2v_F^2(k_{x2}+ik_y)^2}{(E-V_1)^2}\right]e^{2ik_{x2}x},\quad \mathrm{Re}(k_{x2}),\\ \dfrac{|a|^2}{2}\left[1-\dfrac{\hbar^2v_F^2(k_{x2}+ik_y)^2}{(E-V_1)^2}\right]e^{2ik_{x2}x}+a^*b+ab^*\\ \quad+\dfrac{|b|^2}{2}\left[1-\dfrac{\hbar^2v_F^2(k_{x2}-ik_y)^2}{(E-V_1)^2}\right]e^{-2ik_{x2}x},\quad \mathrm{Im}(k_{x2}).\end{cases}\tag{7}$$

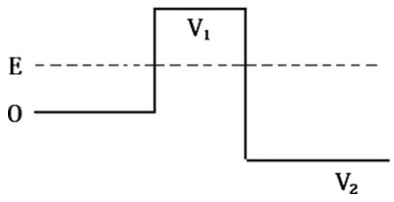

Fig. 1. A schematic diagram for an asymmetric square barrier in graphene.