Physica B 407 (2012) 1725–1733

Contents lists available at SciVerse ScienceDirect

Physica B

journal homepage: www.elsevier.com/locate/physb

Disordered elastic systems and one-dimensional interfaces

E. Agoritsas [a,*], V. Lecomte [b], T. Giamarchi [a]

[a] DPMC-MaNEP, University of Geneva, 24 Quai Ernest-Ansermet, 1211 Geneva 4, Switzerland
[b] Laboratoire Probabilités et Modèles Aléatoires, UMR CNRS 7599, Universités Paris VI et Paris VII, Site Chevaleret, 175 rue du Chevaleret, 75013 Paris, France

ARTICLE INFO

Available online 9 January 2012

Keywords:
Disordered elastic systems
Interfaces
Glassy phenomena

ABSTRACT

We briefly introduce the generic framework of disordered elastic systems (DES), giving a short 'recipe' of a DES modeling and presenting the quantities of interest in order to probe the static and dynamical disorder-induced properties of such systems. We then focus on a particular low-dimensional DES, namely the one-dimensional interface in short-ranged elasticity and short-ranged quenched disorder. Illustrating different elements given in the introductory sections, we discuss specifically the consequences of the interplay between a finite temperature $T>0$ and a finite interface width $\xi>0$ on the static geometrical fluctuations at different lengthscales, and the implications on the quasistatic dynamics.

Contents

1. Introduction

Could some features of experimental systems as dissimilar at a microscopic level, as superconductors, magnets, ferroelectrics, fluids, paper, or two-dimensional electron gases, be described by the same equations at a macroscopic level? All those systems may actually display emergent structures such as *interfaces* (e.g. ferroelectric [1–3] or ferromagnetic [4–6] domain walls, contact line in wetting experiments [7] or propagating cracks in paper and thin materials [8]) or *periodic systems* (typically vortex lattices in type-II superconductors [9], classical [10] or quantum [11] Wigner crystals, or electronic crystals displaying charge or spin density waves [12,13]).

* Corresponding author.
E-mail address: Elisabeth.Agoritsas@unige.ch (E. Agoritsas).

0921-4526/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.physb.2012.01.017

One can either describe them using *ab initio* predictions combined to a Landau approach, where two phases compete with each other at their common boundary (the complexity of a numerical approach increasing considerably with the system size), or rather take a radically opposite point of view by skipping the specific microphysics and focusing exclusively on the boundary, defined by the shift of the order parameter. Such an emergent structure can then be described as a fluctuating manifold or periodic system supported by a disordered underlying medium, in the generic framework of *disordered elastic systems* (DES).

Thereafter we recall briefly the basic features of DES by giving first a short recipe of a DES model based on two competing physical ingredients: elasticity and disorder, blurred by thermal and/or quantum fluctuations. Then we list the main observables of interest in order to probe the disorder-induced metastability present in those systems, and to address the two main questions which arise regarding their resulting glassy properties: what can we learn by the study of their statics *versus* their dynamics, first via the characterization of their geometrical fluctuations and secondly via their response to an external force?

Indeed, from the 1970s' and Larkin's work [14], we know that there could not exist a perfectly ordered solid in the presence of disorder, so how does the addition of disorder change the nature of a pure system? In the last section, we focus on a particular low-dimensional DES and study the static geometrical fluctuations of a one-dimensional interface, via an analysis of the interplay between thermal fluctuations and a finite width of the interface in its roughness.

Those short notes are not meant to be exhaustive, but rather to give a pedagogical and somehow practical introduction to the field, aimed at theoreticians but also at experimentalists who might be interested in DES modeling. We focus essentially on the case of *interfaces*, but most concepts can be extended to *periodic systems*, and more details and references can be found for example starting from the existing reviews [13,15,16].

2. DES modelling: a recipe

In the generic framework of DES, very few physical ingredients are required in a minimal version of such a model. Thereafter we briefly sketch their concrete implementation for interfaces, but those considerations remain valid for periodic systems [17].

2.1. Dimensionality and class of DES

First of all one has to identify the *dimensionality* of the system (d being the internal dimension of the system, m the number of its transverse components, and D the dimension of its embedding physical space) and whether it is a *manifold* ($d+m=D$ with $d=D-1$ for interfaces) or a *periodic system*.

For example, a 1D interface and a single vortex are manifolds respectively with ($d=m=1, D=2$) and ($d=1, m=2, D=3$), whereas an Abrikosov vortex lattice and a 3D Wigner crystal are periodic systems with ($d=D=3$, $m=2$) and ($d=m=D=3$). Note that the dimensionality and the class of DES might actually change with respect to some parameters of the system, as it can been addressed experimentally e.g. for ferromagnetic domain walls (1D to 2D interfaces crossover) [18] or for vortices in superconductors (vortex lattice to individual vortices).

2.2. Physical space of coordinates ($\boldsymbol{z}$,$\boldsymbol{x}$)

The description of the physical space embedding the DES can then be split into two sets of coordinates: $\mathbf{z} \in \mathcal{D}_z$ denotes the *internal* coordinates of the system (e.g. the position along a polymer or a given point in a lattice), and $\mathbf{x} \in \mathcal{D}_x$ its *transverse* coordinates.

For an analytical treatment of interfaces, we typically assume that they live in an *infinite* and *continuous* physical space so $\mathcal{D}_z \times \mathcal{D}_x$ is taken as $\mathbb{R}^d \times \mathbb{R}^m$. However, a physical or numerical realization of such a system is always supported by a *microscopically discrete* sublattice of parameter $1/\Lambda$, ultimately the crystal in a solid, and moreover lives in a *finite* box of typical size L with boundary conditions which have to be defined (possibly periodic or free). So in the comparison between analytical predictions and experimental or numerical results, corrections due to finite size effects and to the translation from the discrete to the continuous limits are *a priori* expected.

Once the disorder is averaged out, a translational space-invariance is recovered, suggesting a description in Fourier space q with an ultra-violet $1/\Lambda$ and an infra-red $1/L$ cutoffs, which are always present in physical DES. However, from an analytical point of view, they are either irrelevant and thus skipped, or they are conveniently reintroduced in order to cure non-physical divergences in computations.

2.3. Univalued displacement field $\mathbf{u_z}$

In the absence of disorder, an elastic system tends to minimize its distortions, thus would typically be flat for an interface or characterized by a single reciprocal vector for periodic systems. A given configuration of a DES is characterized by a *univalued displacement field* $\mathbf{u_z} \in \mathcal{D}_x$ with respect to such an equilibrium configuration (e.g. flat or periodic) of the pure system as illustrated in Fig. 1.

The definition of this reference configuration is actually crucial but potentially tricky in experiments, in particular in certain cases where the equilibrium configuration is not a straight line. For example, how to define unambiguously the center and the mean radius of a 'dotted' ferromagnetic domain if it is far from being perfectly circular? Defects such as overhangs and bubbles for interfaces, or topological defects in periodic systems are still missing in the above DES description, since they hinder the definition of a univalued displacement field $\mathbf{u_z}$, at the core of

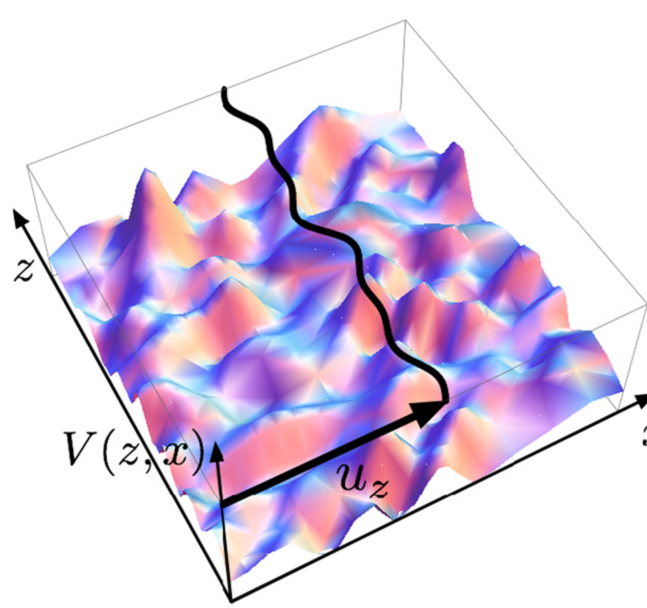

Fig. 1. Definition of the displacement field $\mathbf{u_z}$ for a 1D interface ($d=m=1$) superimposed over its surrounding smooth random potential $V(\mathbf{z},\mathbf{x})$ (in a weak disorder limit). $\mathbf{u_z}$ is univalued only in the absence of bubbles or overhangs for interfaces.