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Josephson effect in graphene SNS junction with a single localized defect

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ABSTRACT

Imperfections change essentially the electronic transport properties of graphene. Motivated by a recent experiment reporting on the possible application of graphene as junctions, we study transport properties in graphene-based junctions with single localized defect. We solve the Dirac–Bogoliubov–de-Gennes equation with a single localized defect superconductor–normal(graphene)–superconductor (SNS) junction. We consider the properties of tunneling conductance and Josephson current through an undoped strip of graphene with heavily doped s-wave superconducting electrodes in the limit $l_{def} \ll L \ll \xi$. We find that spectrum of Andreev bound states are modified in the presence of single localized defect in the bulk and the minimum tunneling conductance remains the same. The Josephson junction exhibits sign oscillations.

1. Introduction

Recent exciting developments in transport experiments on graphene have stimulated theoretical studies of superconductivity phenomena in this material, which has been recently fabricated [1,2]. A number of unusual features [3] of superconducting state have been predicted [4,5] which are closely related to the Dirac-like spectrum of normal state excitation [6–9]. In particular, the unconventional normal electron dispersion [10] has been shown to result in a nontrivial modification of Andreev reflection and Andreev bound states in Josephson junctions [11] with superconducting graphene electrodes [12,13].

Other interesting consequences of the existence of Dirac-like quasiparticles can be understood by studying superconductivity [14] in graphene [15–19]. It has been suggested that superconductivity can be induced in graphene layer in the presence of a superconducting electrode near it via proximity effect [20–22].

In this work, we study Josephson effect and find bound state in graphene [23] for tunneling SNS junction with the presence of a single localized defect [24]. In this study, we shall concentrate on SNS junction with normal region thickness $L \ll \xi$, where ξ is the superconducting coherence length, and width W which has an applied gate voltage U across the normal region [25,26]. In the frame of the limit $l_{def} \ll L \ll \xi$ considered by Kulik we investigate tunneling conductance in SNS junctions with the presence of a single localized defect and find that Andreev levels are modified, the minimum tunneling conductance remains the same [27–29].

2. Josephson effect in superconductor/normal(graphene)/superconductor junctions with a single localized defect

We consider a SNS junction with a single localized defect which is involved in a graphene sheet of width W lying in the x–y plane extends from $x=-L/2$ to $L/2$ while the superconducting region occupies $|x|>L/2$ (see Fig. 1). The SNS junctions can then be described by the Dirac–Bogoliubov–de-Gennes (DBdG) equations [30]

$$\begin{pmatrix} H_s - E_F + U & \Delta \\ \Delta^* & E_F - U - H_s \end{pmatrix} \psi_s = \varepsilon \psi_s.$$

Here, $\psi_s = (\psi_{As}, \psi_{Bs}, \psi^*_{A\bar{s}}, -\psi^*_{B\bar{s}})$, $\psi = (u_1, u_2, v_1, v_2)$ are the four component wave functions for the electron and hole spinors, the index s denote K or K' for electrons or holes near K and K' points, $\bar{s}$ takes values $K(K')$ for $s=K(K')$, E_F denotes the Fermi energy, A and B denote the two inequivalent sites in the hexagonal lattice of graphene, and the Hamiltonian H_s is given by

$$H_s = -i\hbar v_F[\sigma_x \partial_x + \mathrm{sgn}(s)\sigma_y \partial_y]. \quad (1)$$

In Eq. (1), v_F denotes the Fermi velocity of the quasiparticles in graphene and sgn(s) takes values $\pm$ for $s=K(K')$. The 2×2 Pauli matrices σ_i act on the sublattice index. The excitation energy $\varepsilon > 0$ is measured relative to the Fermi level (set at zero). The electrostatic potential U and pair potential Δ have step function profiles, as in the case of a semiconductor two-dimensional electron gas [31–33]: $\Delta(\mathbf{r}) \to \Delta_0 \exp^{\mp i\phi/2}$, $-U$ for $x \to \pm\infty$. We

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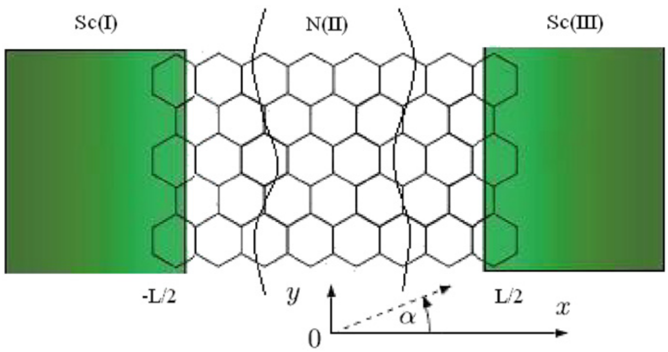

Fig. 1. A graphene undoped ribbon is contacted by two superconducting leads. The charge carriers tunnel from one lead to another via multiple tunneling states formed in the graphene strip. A defect placed inside the strip.

assume non-interacting electrons in the normal region, therefore, $\Delta(\mathbf{r})\equiv 0$, $U=0$ for $|x|<L/2$. The reduction of the order parameter $\Delta(x)$ in the superconducting region on approaching the SN interface is neglected; i.e., we approximate parameter $\Delta(x)$ as we have done it above. As discussed by Likharev [34], this approximation is justified if the weak link has length and width much smaller than ξ. There is no lattice mismatch at the NS interface, so the honeycomb lattice of graphene is unperturbed at the boundary, the interface is smooth and impurity free. Zero magnetic field is assumed.

Solving the DBdG equations, we gain the wave-functions in the superconducting and the normal regions. In region I(III), for the DBdG quasiparticles moving along the $\pm x$ direction with a transverse momentum $k_y=q$ and energy ε, the wave-functions are given by

$$\Psi^{+}=\exp(iqy+ik_s x+\kappa m x)\begin{pmatrix}\exp(-im\beta)\\ \exp(i\gamma-im\beta)\\ \exp(-im\phi/2)\\ \exp(i\gamma-im\phi/2)\end{pmatrix},$$

$$\Psi^{-}=\exp(iqy-ik_s x+\kappa m x)\begin{pmatrix}\exp(im\beta)\\ \exp(-i\gamma+im\beta)\\ \exp(-im\phi/2)\\ \exp(-i\gamma-im\phi/2)\end{pmatrix}.$$

The parameters β, γ, k_0, κ are defined by $\beta=\arccos(\varepsilon/\Delta_0)$, $\gamma=\arcsin[\hbar v_F q/(U_0+E_F)]$, $k_s=\sqrt{(U_0+E_F)^2/(\hbar v_F)^2-q^2}$, $\kappa=(U_0+E_F)\Delta_0\sin(\beta)/(\hbar^2 v_F^2 k_s)$ and $m=\pm$ denotes region I(III), $m=+$ for I and $m=-$ for III correspondingly. Further we assumed that the Fermi wave length λ_F' in the superconducting region much smaller than the wave length λ_F in the normal region and $U_0\gg E_F,\varepsilon$. Since $|q|\le E_F/\hbar v_F$, this regime of a heavily doped superconductor corresponds to the limits $\gamma\to 0$, $k_s\to U_0/\hbar v_F$, $\kappa\to(\Delta_0/\hbar v_F)\sin(\beta)$.

Now we analyze the spectral properties of a graphene ring. Formally we find solutions for graphene ring and then extending ring in the scale, match external boarder of ring with superconducting regions of junction and fix internal part implying which as defect. For that we divide the region II into three areas: a,b,c (see Fig. 2). We solve the DBdG equations for the area b, while area c is the area where placed defect and area a would be extended and matched with superconducting regions. The two valleys $s\pm$ decouple, and we can solve equations separately for each valley, $H_s\psi_s=(\varepsilon+sE_F)\psi_s$, $H_s=H_0+sV(r)\sigma_z$. The term proportional to σ_z in Hamiltonian is a mass term confining the Dirac electrons in the area b. Rewrite the Hamiltonian in the cylindrical coordinates and since H_s commutes with $J_z=l_z+\frac{1}{2}\sigma_z$,

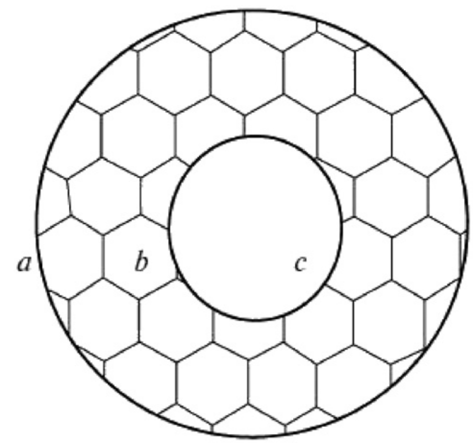

Fig. 2. The normal region II is divided by three areas a, b, c. The DBdG equations are solved for area b and determined by the "infinite mass" boundary conditions induced by $V(r)\to+\infty$ in a and c areas correspondingly.

its electron-eigenspinors ψ_e are eigenstates of J_z [35]

$$\Psi_e(r,\alpha)=\begin{pmatrix}\exp(id(n-1/2)\alpha)J_{d(n-1/2)}(k(\varepsilon)r)\\ \exp(id(n+1/2)\alpha)J_{d(n+1/2)}(k(\varepsilon)r)\end{pmatrix},$$

with eigenvalues n, where n is a half-odd integer, $n=d\frac{1}{2},d\frac{3}{2},\ldots$ and $J_{d(n-1/2)}(k(\varepsilon)r)$ is the Bessel function of $(n-1/2)$ order. In the x–y plane d denotes the moving direction of the correspondent quasiparticle, $d=+$ for the quasiparticle moving toward $x=L/2$ and $d=-$for the quasiparticle moving toward $x=-L/2$ direction correspondingly. Further we are interested in to find zero energy states [22]. In this case the DBdG equations posses a general symmetry with respect to the change in the sign of energy [36]

$$\varepsilon\to-\varepsilon,\quad i\hat{\sigma}_y\hat{u}^*\to\hat{v},\quad i\hat{\sigma}_y\hat{v}^*\to-\hat{u},\tag{2}$$

where we denote $\hat{u}=(u_1,u_2)$ and $\hat{v}=(v_1,v_2)$. Thus, for a set of zero modes $(\hat{u}_i,\ \hat{v}_i)$ enumerated by a certain index i we have

$$\hat{v}_i=i\hat{\sigma}_y\hat{u}_j^*,\quad \hat{u}_i=-i\hat{\sigma}_y\hat{v}_j^*.\tag{3}$$

In the same manner for electron hole-spinors we have the view

$$\Psi_h(r,\alpha)=\begin{pmatrix}-\exp(id(n+1/2)\alpha')J_{d(n+1/2)}(k'(\varepsilon)r)\\ \exp(id(n-1/2)\alpha')J_{d(n-1/2)}(k'(\varepsilon)r)\end{pmatrix},$$

with the definitions

$$\alpha(\varepsilon)=\arcsin[\hbar v_F q/(\varepsilon+E_F)],\tag{4}$$

$$\alpha'(\varepsilon)=\arcsin[\hbar v_F q/(\varepsilon-E_F)],\tag{5}$$

$$k(\varepsilon)=(\hbar v_F)^{-1}(\varepsilon+E_F)\cos(\alpha),\tag{6}$$

$$k'(\varepsilon)=(\hbar v_F)^{-1}(\varepsilon-E_F)\cos(\alpha).\tag{7}$$

The angle $\alpha\in(-\pi/2,\pi/2)$ is the angle of incidence of the electron (having longitudinal wave vector k), and α' is the reflection angle of the hole (having longitudinal wave vector k') [37,38]. To obtain an analytical approximation of the spectrum, we use the asymptotic form of the Bessel functions for large r. This indeed is the desired limit as $rk(\varepsilon)\approx r_{def}k(\varepsilon)\propto r_{def}/L\ll 1$, where r_{def} is the defect radius (the radius of the area c) and determine for all eigenvalues $n=d\frac{1}{2}$. In this limit we impose the "infinite mass" boundary conditions at $y=0,W$, for which $q_n=(n+1/2)\pi/W$ in the area b with $V(r)\to+\infty$ in the a and c areas consequently (see Fig. 3). Half-odd integer values n reflect the π Berry's phase of closed size of a single localized defect in graphene.

To obtain the subgap ($\varepsilon<\Delta_0$) Andreev bound states, we now impose the boundary conditions at the graphene. The wave-functions in the superconducting and normal regions can be