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Localization–delocalization transition in two-dimensional system with correlated disorder

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ABSTRACT

We study the localization properties of the two-dimensional (2D) Anderson model with diagonal correlated disorder named visible vw and non-visible nw sites. Using the exact diagonalization of Anderson Hamiltonian, investigating the behavior of the density of states (DOS), we found that states in the center of the band $E=0$ show critical behavior. The energy levels statistics (ELS) is also examined, we found a crossover of the nearest-neighbor level spacing distribution $P(s)$ from Wigner Surmise distribution at small correlated disorder vw and nw (indicating a metallic behavior) to the Poisson distribution at large correlated disorder vw and nw characteristic for localized states. In addition, an analysis of width M dependence of the reduced localization length λ/M is obtained by the transfer-matrix method (TMM). For all investigated strengths of correlated disorder at $E=0$ and $E=+2$, the reduced localization length λ/M always decreases with increasing width. We found only delocalized states in suggested energy and disorder range.

1. Introduction

The scaling theory proposed by Abrahams et al. [1] in 1979 is significant for the theory of disordered systems and the concept of Anderson localization [2]. The scaling hypothesis means the absence of metal–insulator transition (MIT) in (1D) and (2D) disordered systems, so that with any amount of disorder, all states are localized and the system is an insulator [3,4]. Contrary to this well-established hypothesis, the pioneering experiment in 1994 by Kravchenko et al. [5], showed the existence of metal–insulator transition for two dimensions interacting electron gas at low electronic density.

However, recently some examples of (1D) systems with correlated disorder [6–9] have been found to exhibit a number of extended states. GaAs/AlGaAs superlattices including intentionally correlated disorder (dimer barrier or well potentials) were experimentally realized [10] and delocalization properties were observed by photoluminescence measurements. Furthermore, the existence of a mobility edge separating extended and localized states was confirmed for two-dimensional (2D) random systems with long-range correlated disorder [11–13]. Moreover, numerical studies on (2D) Anderson model with off-diagonal disorder [14,15] led to delocalized states at band center energy $E=0$. It was shown by the transfer matrix method (TMM) that the localization length at this energy diverges [14,16].

The statistical properties of the energy level spectra reflect the character of eigenstates and have been proven to be a powerful tool for characterizing the delocalization [17–21]. On the insulating side of the MIT, one finds that localized states that are close in energy are usually well separated in space. Consequently, the eigenvalues on the insulating side are uncorrelated. There is no level repulsion and the probability of eigenvalues to be close together is high. This is called level clustering and is described by the Poisson statistics. On the other hand, extended states occupy the same regions in space and their eigenvalues become correlated. This results in level repulsion such that the spectrum properties are given by the Wigner Surmise statistics.

Most of the theoretical work on Anderson localization has been based on simple tight binding models, where the diagonal matrix element of the Hamiltonian are independent random variables. The question of statistically correlated matrix elements has not been extensively studied, except in the case of the binary-alloy disorder [22–24] in (1D). These special types of correlated disorder can produce extended states.

In the present work, we investigate numerically the (2D) Anderson model of localization including visible and non-visible correlation on the energy sites which are nonindependent random variables with *vw* or *nw*. The exact model is described in Section 2. We calculate the density of states (DOS) and show the effect of correlation on the states in the band center $E=0$ and use energy level statistics (ELS) to characterize the localization and

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delocalization. We check the system size dependence of the reduced localization length (λ/M) up to width $M=150$ at $E=0$ and $+2$ with different strengths of correlated diagonal disorder. In Section 3, we present our results.

2. Model and method of calculation

The 2D Anderson Hamiltonian is given as [2]

$$H=\sum_{i}^{N}\varepsilon_i\ |i\rangle\langle i|\ +\sum_{i\neq j}^{N}t_{i,j}|i\rangle\langle j| \tag{1}$$

The sites i and j form a simple square lattice of size $N=L\times L$. The onsite energies ε_i are taken to be randomly distributed in the interval $[-W/2,W/2]$ where W is the width of the diagonal correlated disorder and $t_{i,j}$ are the transfer integrals or hopping elements restricted to nearest-neighbors to be randomly distributed in the interval $[c-w/2,c+w/2]$ thus c represents the center and w the width of the off-diagonal disorder. For pure diagonal disorder $t_{i,j}$ are constant ($c=1$ and $w=0$). We will always use periodic boundary conditions.

The disorder correlation is obtained mathematically by defining the i and j state as numbers, if i and j are prime the correlation is named visible vw and if not it is named non-visible nw [25]. A typical configuration of visible correlation for system size $N=40\times 40$ is given in Fig. 1.

We use the Routine DE4CSB [26] to compute the spectrum of H for the diagonalization of sparse matrix. From this eingenvalue, the DOS and the energy level spacing distribution $P(s)$ is calculated, here $s=|E_{i+1}-E_i|/\Delta$ is the energy separation of consecutive eigenvalues E_i divided by the mean level separation Δ.

The transfer matrix method (TMM) [27,28] is used to study the states localization properties in disordered systems to calculate the decay lengths of wave function on strips (quasi-1D systems) of width M and length $L\times M$

$$\begin{pmatrix}\Psi_{n+1}\\ \Psi_n\end{pmatrix}=\begin{pmatrix}[t^{||}_{n+1}]^{-1}(E-\varepsilon_n-H_\perp) & -[t^{||}_{n+1}]^{-1}t^{||}_n\\ 1 & 0\end{pmatrix}\begin{pmatrix}\Psi_n\\ \Psi_{n-1}\end{pmatrix}=T_n\begin{pmatrix}\Psi_n\\ \Psi_{n-1}\end{pmatrix} \tag{2}$$

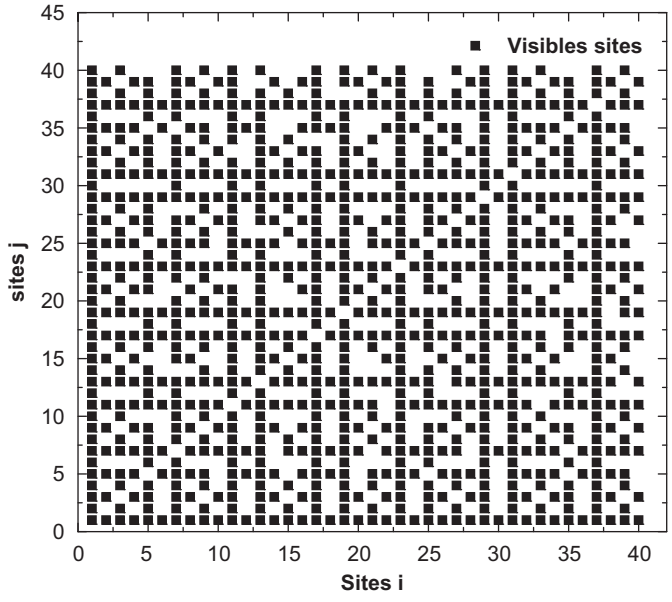

Fig. 1. Visibles sites of square lattice for size $N=40\times 40$.

Here $\Psi_n=(\Psi_{n,1},\Psi_{n,2},\ldots,\Psi_{n,M})^T$ is the wave function at all sites of the n layer, $\varepsilon_n=diag\ (\varepsilon_{n,1},\varepsilon_{n,2},\ldots,\varepsilon_{n,M})$ is the energy, $H_\perp$ the hopping hamiltonian in the n-th layer, 1 and 0 are unit and zero matrices, $t^{||}_{n+1}=\mathrm{diag}(t^{||}_{n,1}\ldots,t^{||}_{n,M})$ is the diagonal matrix represents the hopping elements connecting the (n-1) layer with (n) layer and T_n is the transfer matrix. The evolution of the wave function is given by the product of the transfer matrices $\tau_k=T_k\times T_{k\text{-}1}\times\cdots\times T_1$

The eigenvalues of $\exp[^+_-\gamma_i(M)]$ of $\lim_{k\to\infty}(\tau^t_k\times\tau_k)^{1/2k}$ exist and the smallest Lyapunov exponent $\gamma_{\min}>0$ corresponds to the largest localization length λ at energy E:

$$\lambda=\frac{1}{\gamma_{\min}} \tag{3}$$

3. Results and discussion

In this section we calculate the density of states (DOS) in order to study the effect of correlation on the states in the band center $E=0$ and use energy level statistics (ELS) to characterize the delocalization transition for correlated disorder nw and vw. The system size is $N=40\times 40$. The DOS and the energy level spacing distribution $P(s)$ are obtained by averaging over 500 samples and energy interval $\Delta E=0.01$.

In Fig. 2, we plotted the density of states (DOS). It is clearly seen that the DOS presents a singularity at the band center $E=0$ which is more pronounced for non-visible diagonal disorder ($nw=2$, $vw=0$), because the concentration of non-visible sites is less than that of visible sites, and suppressed when the system is completely disordered ($vw=nw=2$). This is the major effect of the correlation on the DOS. In the usual Anderson model with diagonal disorder, this singularity is quickly suppressed when the disorder strength is increased [14]. It was found by Eilmes et al. that for an uncorrelated disorder, the singularity at $E=0$ is suppressed when the disorder strength is increased [15].

The energy level spacing distribution $P(s)$ has been used [29] as a criterion in distinguishing localized states, which follows the Poisson distribution $P(s)=\exp(-s)$, from extended state which follows the Wigner distribution $P(s)=(\pi/2)s\exp(-(\pi/4)s^2)$.

Our numerical studies of $P(s)$ show a crossover for weak visible disorder from Wigner distribution (indicating a metallic behavior) to Poisson distribution for strong visible disorder (see Fig. 3a). The same behavior is observed for non-visible disorder (see Fig. 3 b). However, in the presence of visible and non-visible disorder, $P(s)$

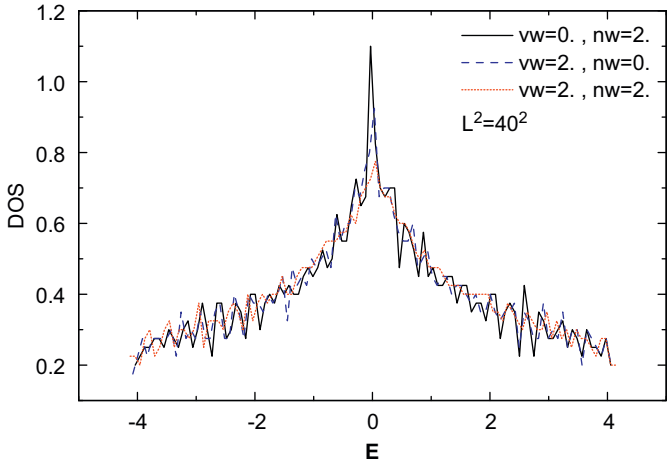

Fig. 2. (Color online) Density of states for purely diagonal disorder.