Physica B 404 (2009) 1766–1770

Contents lists available at ScienceDirect

Physica B

journal homepage: www.elsevier.com/locate/physb

Experimental investigation of the acoustic pressure in cavity of a two-dimensional sonic crystal

Liang-Yu Wu [a], Lien-Wen Chen [a,*], Chia-Ming Liu [b]

[a] Department of Mechanical Engineering, National Cheng Kung University, Tainan, Taiwan
[b] Nano Technology Research Center, Industrial Technology Research Institute, Hsinchu, Taiwan

ARTICLE INFO

Article history:
Received 19 December 2008
Received in revised form
12 February 2009
Accepted 15 February 2009

PACS:
63.20−e
43.35.+d
43.40.+s

Keywords:
Sonic crystal
Resonant cavity
Point defect

ABSTRACT

The experimental observation of the pressure in the cavity of a 2-D sonic crystal is presented. The sonic crystal consists of the polymethyl methacrylate cylinders in air background. The full band gap and defect mode are demonstrated experimentally and compared with the calculating results. The spectra in the cavity are measured for different sizes of the sonic crystal. The pressure in the cavity and quality factor of the cavity can be enhanced as the sonic crystal is enclosed by the cylinders. The numerical results are confirmed by the experiment.

1. Introduction

The propagation of acoustic and elastic waves in the periodic media known as sonic crystals is a problem of increasing interest in recent years [1–14]. Such artificial crystals can exhibit band gaps in which sound and vibration are all forbidden, giving rise to prospective applications such as elastic/acoustic filters, noise/vibration isolations, as well as improvement in transducer design. The existence of the elastic/acoustic wave band gaps is significant to better understand localization in inhomogeneous media. One particularly interesting aspect of sonic crystals is the possibility of creating crystal defects to confine the elastic/acoustic waves in localized modes [5–14]. Because of locally breaking the periodicity of the structure, the defect modes can be created within the band gaps, which are strongly localized around the local defect.

The propagation of sound through a 1-D periodic array of water and perspex plates was studied theoretically and experimentally [5]. The transmission coefficient of various finite structures was calculated and measured as a function of frequency. Narrow pass bands can be placed in the band gaps by introducing a simple defect into the crystal. Moreover, for 2-D sonic crystals, point defects can act as acoustical filters and line defects can be used as a wave-guide [6–13]. Point defect modes of acoustic wave in 2-D square arrays of water rods in mercury host were studied [9–11]. The defects are created by three kinds of geometry, that is, square defect, circular defect, and rectangular defect [10]. The numerical results show that for both the square and circular defects, the defect modes are only related to the defect filling fraction, but not with the geometry of the defects (circular or square) as well as the orientations of the square defect. For the rectangular defect, the defect modes could be tuned by varying the ratio of the edge width. The symmetric property and coupling efficiency of the defect modes in a 2-D sonic crystal had been studied by calculating band structures, field distributions, and transmission coefficients of the defect modes [11]. The results have shown that the coupling efficiency of defect modes strongly depends on the thickness of the composite layers surrounding the cavity. Furthermore, acoustic point defect states in the 3-D simple-cubic arrays of water spheres embedded in a mercury host had been studied [14]. Their results are similar to 2-D case, namely, the defect band is only related to the defect filling fraction, not the geometry of defect (sphere or cube).

For the experiments, several cases of the 2-D sonic crystal were reported [3,4,12,13]. The band gaps had been measured for the rigid rods in air [3] and steel cylinders immersed in water [4]. Khelif et al. had investigated experimentally the acoustic band-gap effect in a 2-D sonic crystal composed of steel rods immersed in water [12]. The full band gap and defect modes in the band gap

* Corresponding author. Tel.: +886 6 2757575x62143; fax: +886 6 2352973.
E-mail address: chenlw@mail.ncku.edu.tw (L.-W. Chen).

0921-4526/$ - see front matter
doi:10.1016/j.physb.2009.02.025

had been observed by removing one or more rods from the lattice. The result of the experimental development of the wave-guide with sharp bend to a pair of coupled wave-guides constructed in a sonic crystal had been reported [13]. The intensities of the sound waves that traveled along the wave-guide and the leaky waves which propagated straight without turning the bend had been measured. But, the pressure in the point defect of a sonic crystal has not been experimentally studied.

In this paper, we study the point defect in the 2-D sonic crystals composed of polymethyl methacrylate (PMMA) cylinders with square array embedded in air background. The calculations of band structures are based on the plane wave expansion (PWE) and the defect mode is determined by using the supercell calculation. The pressure in the point defect is dependent on incident frequencies, and it reaches the maximum at the resonant frequency. The point defect can act as the resonant cavity. We find that the quality factors of the cavity and pressures in the cavity at the resonant frequency can be enhanced by using the cylinders enclosed in an original 5×5 sonic crystal. Experimental measured data are presented and compared with the numerical and simulated results.

2. Numerical modelling

A 2-D periodic system that comprises rods immersed in fluid background is studied. Since the fluid cannot support the propagation of a transverse wave, the longitudinal wave is only allowed to propagate. The wave propagation along such a sonic crystal is predominant only in the fluid. It is a good approximation to consider the solid rods as fluid inclusions with very high stiffness and specific mass. Then, the wave equation is simplified as follows [2]:

$$(C_{11})^{-1}\frac{\partial^2 \boldsymbol{p}}{\partial t^2} = \nabla\cdot(\rho^{-1}\nabla\boldsymbol{p}), \tag{1}$$

where $\boldsymbol{p}$ is the pressure, ρ is the mass density, $C_{11}=\rho c_l^2$ is the longitudinal elastic constant, and c_l is the sound speed. By applying the Bloch theorem and Fourier series, the eigenvalue equation was obtained as follows [2]:

$$\sum_{\mathbf{G}'\neq\mathbf{G}} F(\mathbf{G}-\mathbf{G}')[\varDelta(\rho^{-1})(\mathbf{K}+\mathbf{G})\cdot(\mathbf{K}+\mathbf{G}') - \varDelta(C_{11}^{-1})\omega^2]\boldsymbol{p}_{\mathbf{K}}(\mathbf{G}') + [\overline{\rho^{-1}}|\mathbf{K}+\mathbf{G}|^2 - \overline{C_{11}^{-1}}\omega^2]\boldsymbol{p}_{\mathbf{K}}(\mathbf{G}') = 0, \tag{2}$$

where $\mathbf{G}$ is the 2D reciprocal lattice vectors, and $\mathbf{K}$ is a 2D Bloch vector. $\omega(\mathbf{K})$ and $\boldsymbol{p}_{\mathbf{K}}(\mathbf{G})$ are the eigenvalues and eigenvectors, respectively.

For the rods with radius r_0 in the system, structure factor $F(\mathbf{G})$ can be obtained as below [2]:

$$F(\mathbf{G}) = 2f\frac{J_1(\mathbf{G}r_0)}{\mathbf{G}r_0}, \tag{3}$$

where $f=\pi r_0^2/a_0^2$ is the filling fraction of rods with lattice constant a_0 is the square lattice, and $J_1(x)$ is the Bessel function of the first kind of order one. The band structures are obtained by solving the eigenvalue equation, Eq. (2). Moreover, the supercell calculation can be employed to obtain the defect band for the sonic crystal with a defect [9].

The *COMSOL Multiphysics*® software is used to simulate the acoustic wave propagation in the sonic crystals [15]. The equation used to analyze the acoustic wave problems is expressed as

$$-\nabla\cdot\frac{\nabla\boldsymbol{p}}{\rho} = \frac{\omega^2}{\rho c_l^2}\boldsymbol{p}. \tag{4}$$

By solving Eq. (4), the pressure field in the sonic crystal can be obtained.

3. Numerical results

We consider a 2-D sonic crystal consisting of PMMA cylinders in air background with square lattice. The cylinders have a radius $r_0 = 17.5$ mm. The lattice constant is $a_0 = 39$ mm. This results in a filling ratio of approximately 63%. The material parameters employed in the calculations and simulations are $\rho_{\mathrm{air}} = 1.2\,\mathrm{kg/m^3}$, $\rho_{\mathrm{PMMA}} = 1190\,\mathrm{kg/m^3}$, $c_{\mathrm{air}} = 343\,\mathrm{m/s}$, and $c_{\mathrm{PMMA}} = 2694\,\mathrm{m/s}$. Fig. 1(a) shows the band structure calculated by the PWE method. We find that the partial band gap is from 2.3 to 6.22 kHz and 3.69 to 6.22 kHz for the Γ–X direction and Γ–M direction, respectively. There exists a full band gap between 3.69 and 6.22 kHz. The point defect is created by removing a single rod from the middle of the perfect periodic structure. In order to calculate the band structure of the sonic crystal with a point defect, the PWE method and supercell calculation are adopted. By using two methods, the band structure of 5×5 supercell with a point defect is shown in Fig. 1(b). We can see that there exist two defect bands in the band gap. The defect bands can act

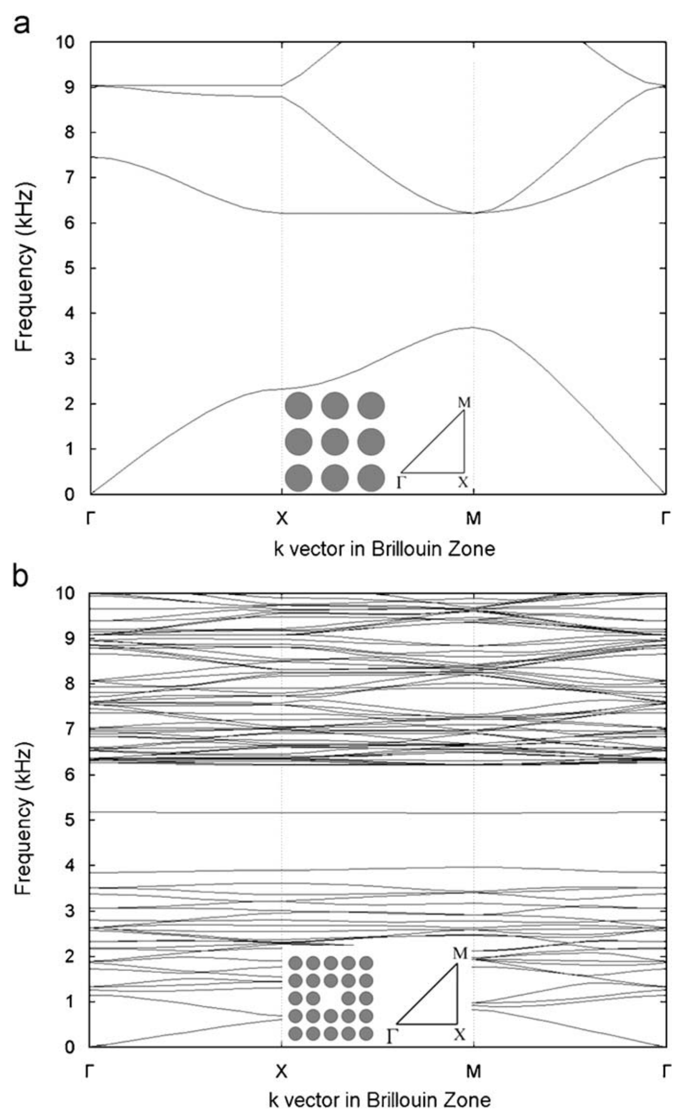

Fig. 1. (a) The band structure of the 2-D sonic crystal consisting of the PMMA cylinder in air background. The inset shows the sonic crystal arranged in a square lattice and the irreducible Brillouin zone. (b) The band structures of the 5×5 supercell with a point defect. The inset shows the 5×5 supercell arranged in a square lattice and the irreducible Brillouin zone. A rod is removed from the middle of the supercell.