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Dynamic conductivity of composites of fractal structure

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ABSTRACT

We have studied the dynamic conductivity of a composite consisting of well- and weak-conducting components with random fractal structure. In order to calculate effective properties of composite medium, we used hierarchic structure model and innovative iterative averaging method based on renormalization group transformations idea. Our results show, that the behavior of a composite over a magnetic field become even more complicated.Unusual peaks and oscillations appear in frequency dependencies of effective conductivity, permittivity and other properties. We discuss the influence of fractal parameters of the composite structure on such unusual behavior of effective properties.

1. Introduction

The description of charge transfer in a magnetic field is one of the most intricate problems of the transfer theory in heterogeneous media. There exist some methods that allow calculating galvanomagnetic properties of inhomogeneous media, such as aggregates, suspensions, colloids, etc. These methods are mostly based on approximate Clausius–Masotti-type methods, Bruggeman-type models of effective medium, perturbation methods with such parameters as a small field, small concentration, small difference of properties [1,2]. All these methods more or less well describe the inhomogeneous medium in limited variation intervals of concentration, field and other parameters.

Despite a great deal of such theoretical efforts, these methods are not in clear agreement with the percolation theory and experimental results for the nanocomposites, consisting of small (1–10 nm) metal inclusions set implanted in a dielectric material [3,5]. The analysis becomes even more difficult when we examine the dependence of galvanomagnetic properties on frequency (dynamic conductivity, Hall dynamic coefficient).

The peculiarity of heterogeneous percolation media with chaotic structure is the geometric phase transfer which appears when the percolation cluster is formed [5–7]. This percolation cluster is a fractal set.

The main feature of fractal behavior is dependence of the properties, C, on the linear scale, L: $\tilde{L}^{\alpha}$ where α is a constant.

This dependence is the consequence of self-similarity (scaling) of fractal structure. In real media, it is usually limited by the area of intermediate asymptotic which is defined as $l_0 \leq L \leq \xi$. Here, l_0 is the microscopic constant, e.g. constant of the lattice; ξ is the correlation length, i.e. the length determining the scale where properties of a heterogeneous medium depend on the configurations — on the distribution of heterogeneity.

In the scale area $L \gg \xi$, i.e. in the standard conditions, the microheterogeneous medium becomes a homogeneous one. The self-similarity and dependence on the scale disappear. In such conditions, the medium can be characterized with the effective properties.

Therefore, in media with fractal structure, there are two kinds of conditions: the fractal and standard behavior.

The self-similarity is an important feature of fractals. They are geometric objects in which small parts are similar to the whole. In other words, fractals are geometric objects — superensembles consisting of hierarchically subordinated ensembles which, in their turn, consist of a set of sub-ensembles and so on.

The physical sense of the conception of hierarchical structure of a material is based on the fact that small-scaled structure of the material cannot take part in the energetic exchange until the average energy absorbed by the system reaches the certain level — the level of excitation on the given scale.

It is worth mentioning that media with fractal structure have properties which are different from those of such heterogeneous media as gases, amorphous solid matter and ordinary composites [5].

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For instance, fractal structure causes the effect of gigantic combination dissipation (GCD), which is a huge increase of the effective section (10^5–10^6 times) of light combination diffusion. The GCD effect is observed on island films, in colloids with microscopic metal particles (about 100–1000 Å), and in hydrosols of precious metals [8–17]. It cannot be explained by classical theory.

Besides that, a number of gigantic effects in unregulated media have been discovered in the past decades, in particular the Faraday gigantic effect [8,18].

Unusual properties of media with fractal structure, including galvanomagnetic properties, are caused by the fact that conductors of completely the same shape and size will have different resistance at the hierarchic meso-levels. The conductor sizes of each heterogeneity are distributed randomly at the meso-level. Each conductor is characterized by individual dependence of its properties on the outer electric field (also on the magnetic field). The hierarchic structure of a material also gives an opportunity to consider each hierarchic level as really existing and having physical parameters, for example conductivity. Under certain conditions, these conductors may have negative values on meso-levels (the local change of the direction of the charge carrier stream occurs), which leads to huge fluctuations of the electric fields [19]. These fluctuations reveal themselves especially on scales $l_0 \leq L \leq \xi$ near the percolation threshold.

Direct experimental estimations of microstructure and galvanomagnetic properties of materials with chaotic structure are rather laborious and expensive. In these conditions, calculation approaches allowing to carry out virtual examination of the structure and properties of heterogeneous media are of crucial importance. They allow to use a computer for the calculations that take into consideration structural peculiarities of such materials on meso-levels.

The fractal model of the structure of a heterogeneous medium and the iteration method of averaging the physical properties are shown below. The analysis of the influence of hierarchic fractal structure of a metal–dielectric composite on the dynamic conductivity of heterogeneous percolation media will be carried out using the above method by $B \neq 0$.

The calculations will be done assuming that the length of the wave (of forced oscillations) falling upon the sample is much larger than the typical size of the heterogeneity (insertion). In this case, the composite may be replaced with a quasi-homogeneous medium possesses the effective properties.

2. Spectroscopic characteristics of the charge transfer in the outer magnetic field

When electromagnetic waves are spreading in a heterogeneous medium with applied magnetic field, a number of magneto-optic effects appear (those of Faraday, Kerr, Feucht) [20]. The dynamic conductivity of a medium in a magnetic field defines its magneto-optic properties and manifests itself when the optical properties of the medium alter in appearance of the magnetic field. The dynamic tensor of the magnetic conductivity is introduced to describe the magneto-optic properties. The tensor's components can be obtained from the description of the electronic properties of the transfer in a solid matter. It is convenient to use the complex conductivity, σ, and complex permittivity, ε, to analyze the dynamic tensor spectral properties. The connection between complex conductivity σ and complex permittivity ε of a material medium can be defined from the equation for the current density on the basis of Maxwell [20,21].

$$j(t) = \sigma E + \varepsilon \frac{\partial D(t)}{\partial t} \tag{1}$$

After applying Fourier transform, it can be written in two forms:

$$\tilde{j}(\omega) = \tilde{\sigma}(\omega)\tilde{E}(\omega) \tag{2}$$

$$\tilde{j}(\omega) = i\omega\tilde{\varepsilon}(\omega)\tilde{E}(\omega) \tag{3}$$

where

$$\tilde{\sigma}(\omega) = \sigma + i\omega\varepsilon \tag{4}$$

i.e.

$$\tilde{\sigma}(\omega) = i\omega\tilde{\varepsilon}(\omega) \tag{5}$$

Thus, Ohm's law for the dynamic magnetic conductivity can be written as (see Appendix A)

$$\tilde{j}(\omega) = \tilde{\sigma}(\omega)\tilde{E}(\omega) \tag{6}$$

here $\tilde{\sigma}(\omega)$ is the dynamic magnetic conductivity tensor

$$\tilde{\sigma}(\omega) = \begin{pmatrix} \sigma_{xx}(\omega) & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy}(\omega) & 0 \\ 0 & 0 & \sigma_{zz}(\omega) \end{pmatrix} \tag{7}$$

The complex components of conductivity tensor $\tilde{\sigma}(\omega)$ presented by spectroscopic features can be defined as [21]

$$\sigma_{xx}(\omega) = \sigma_{yy}(\omega) = i\omega\varepsilon_0 \left[1 - \frac{\omega_p^2(\omega - i\omega_\tau)}{\omega(\omega - i\omega_\tau)^2 - \omega_c^2}\right] \tag{8}$$

$$\sigma_{xy}(\omega) = -\sigma_{yx}(\omega) = \frac{\varepsilon_0 \omega_p^2 \omega_c}{(\omega - i\omega_\tau)^2 - \omega_c^2} \tag{9}$$

$$\sigma_{zz}(\omega) = \frac{\varepsilon_0 \omega_p^2}{\omega - i\omega_\tau} \tag{10}$$

The spectroscopic parameters are connected with the ohmic conductivity σ, Hall conductivity $1/(R \cdot B)$, Hall mobility μ, Hall angle $\tan\Phi_H = -\mu B$, $\mu = \sigma R$, where R is Hall coefficient, as

$$\omega_\tau = \frac{1}{\tau} \tag{11}$$

where ω_τ is the frequency of collisions, ω_p the plasma frequency, and ω_c the cyclotron frequency.

Sometimes it is convenient to describe magneto-optic effects in terms of the tensor of magnetic resistance with the components $\rho_{xx}(\omega), \rho_{yy}(\omega), \rho_{zz}(\omega), \rho_{xy}(\omega), \rho_{yx}(\omega)$ which is inverse to the tensor of the dynamic magnetic conductivity (8)–(10):

$$\rho_{xx}(\omega) = \rho_{yy}(\omega) = \rho_{zz}(\omega) = \frac{(\omega - i\omega_\tau)}{\varepsilon_0 \omega_p^2} \tag{12}$$

$$\rho_{xy}(\omega) = -\rho_{yx}(\omega) = -\frac{\omega_c}{\varepsilon_0 \omega_p^2} \tag{13}$$

It follows from Eqs. (8)–(10): if $B = 0$, $\omega_c^* = 0$, we have the usual dynamic conductivity; if $\omega = 0$, the non-diagonal components describe Hall effect:

$$j = (0, j_y, 0) \tag{14}$$

In the dynamic case $\omega \neq 0$, the non-diagonal members σ_{xy} define Faraday effect in Eq. (7), which can be considered as the dynamic Hall effect.

When building the fractal model of heterogeneous medium chaotic structure, the randomly distributed set of bonds of two sorts is considered [5,22,23].

It is assumed that each n-th bond of the set has the collection of Hall properties $(\sigma_{xx}^{(n)}(\omega), \sigma_{xy}^{(n)}(\omega))$ consisting of frequency-dependent conductivities $\sigma_{xx}(\omega)\sigma_{xy}(\omega)$. Then two configurations of the