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d-Wave to s-wave to normal metal transitions in disordered superconductors

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ABSTRACT

We study suppression of superconductivity by disorder in d-wave superconductors, and predict the existence of (at least) two sequential low-temperature transitions as a function of increasing disorder: a d-wave to s-wave, and then an s-wave to metal transition. This is a universal property of the system which is independent of the sign of the interaction constant in the s-channel.

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Generally the order parameter in superconductors is a function of two coordinates and two spin indices. Classification of possible superconducting phases in crystalline materials was done in Refs. [1,2]. A majority of low-T_c crystalline superconductors have a singlet order parameter with s-wave symmetry. It does not change its sign under rotation, and in the isotropic case it can be approximated by a complex number $\Delta^s(\mathbf{r}) = \Delta(\mathbf{r},\mathbf{r})$. However, over the last decades a number of superconductors have been discovered in which the order parameter changes sign under rotation. A notable example is HTC superconductors, where in the absence of disorder the order parameter has singlet d-wave symmetry [3,4]: $\Delta(\mathbf{r}-\mathbf{r}')$ changes sign under rotation by $\pi/2$, and consequently $\Delta(\mathbf{r},\mathbf{r}) = 0$. This means that the Fourier transform $\Delta(\mathbf{k})$ changes its sign under a $\pi/2$ rotation as well, as is shown schematically by the rosettes in Fig. 1. Since the sign of $\Delta(\mathbf{k})$ in crystalline d-wave superconductors depends on the direction of the wave vector $\mathbf{k}$, they are much more sensitive to disorder than s-wave superconductors: at temperature $T=0$, d-wave superconductivity gets destroyed when the electron mean free path l is of the order of the zero temperature coherence length in a pure superconductor, $l \sim l_0 = 1.78\xi_0 \gg 1/k_F$. Here k_F is the Fermi wavelength. This is in contrast with the case of s-wave superconductors, where according to the Anderson theorem the superconductivity is destroyed at much higher level of disorder, when $l \sim 1/k_F$. The fate of the d-wave superconductors at $l < \xi_0$ depends on the sign of the interaction constant λ_s in the s-wave channel. If the interaction λ_s in the s-wave channel is attractive, but weaker than the attraction in the d-wave channel $|\lambda_s| < |\lambda_D|$, then at weakdisorder, ($l > \xi_0$), the superconducting order parameter has d-wave symmetry, while at $l < \xi_0$ the disorder destroys the d-wave superconductivity and the system undergoes a phase transition into an s-wave superconducting state. (See, for example, Ref. [5].)

In this article we consider a more interesting case, in which the interaction in the s-channel is repulsive at strong enough disorder $1/k_F \ll l \ll \xi_0$ the system is in normal state. We predict at least two low-temperature phase transitions: a d-wave to s-wave, and then an s-wave to normal metal transition. Qualitatively the phase diagram of disordered d-wave superconductors is shown in Fig. 1. Let us first discuss the definition of s- and d-symmetries in bulk disordered systems. Before averaging over random realizations of disorder, the system does not possess any particular spacial symmetry at all. However, in bulk samples, the symmetry is restored upon configuration averaging. We can think of several different definitions of the global symmetry of the order parameter: (a) an operational definition is provided by the result of a phase sensitive experiment, such as the corner SQUID experiment, for example, [3,4]. (b) The quantity $\overline{\Delta(\mathbf{r},\mathbf{r}')}$ can be characterized as having d-wave or s-wave symmetry. Here the over-line stands for the averaging over the sample volume. (c) A globally s-wave component of the order parameter can be defined in terms of the local s-component of the anomalous Green function $\mathscr{F}(\mathbf{r}=\mathbf{r}') \equiv \mathscr{F}^{(s)}(\mathbf{r})$. If we define $P_\pm$ to be the volume fraction of a sample where $F^{(s)}(\mathbf{r})$ has a positive or a negative sign, respectively, then the system has an s-wave component if $(P^+ - P^-) \neq 0$. These definitions may be not equivalent under all circumstances. However, for the most part, we will deal with the interval of parameters in which all these definitions are approximately interchangeable.

It is important to realize that it is inevitable near criticality to have a situation in which the local pairing in disordered superconductors is "d-wave-like" and yet the global superconductivity has s-wave symmetry. The d-wave to s-wave transition can be understood at the mean field level. The electron mean free path is an average characteristic of disorder. Let us introduce a "local" value of the mean free path $l(\mathbf{r})$ averaged over a size of order ξ_0. In the region of parameters where d-wave superconductivity is

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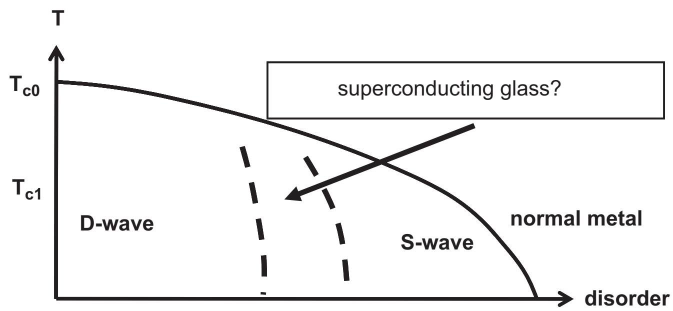

Fig. 1. Schematic phase diagram for the case when d-wave superconducting state is destroyed as a function of increasing disorder strength.

sufficiently suppressed by disorder, the spatial dependence of the order parameter can be visualized as a system of superconducting puddles with anomalously large values of the order parameter, which are connected by Joshepson links through non-superconducting metal. The superconductivity inside the puddles may be enhanced because either the electron interaction constant or the mean free path in the puddles (or both) may be larger than their average values.

Let us assume that the distance between the puddles is larger than both their size and the mean free path. In this case the system is already in a state with the "global s-wave" symmetry. Its origin is illustrated qualitatively in Fig. 2, where a system of superconducting puddles of arbitrary shape embedded into a metal is shown. The order parameter inside the puddles has d-wave symmetry, and the orientation of the gap nodes is assumed to be pinned by the crystalline anisotropy. In a d-wave superconductor, in addition to an overall phase of the order parameter, there is an arbitrary sign associated with the internal structure of the pair wave function. Specifically, we adopt a uniform phase convention such that when the phase of the order parameter $\phi_i = 0$, this implies $\Delta(\mathbf{r}, \mathbf{r}')$ in puddle i is real and has its positive lobes along the y axis and its negative lobes along the x axis.

The inter-puddle Joshepson coupling originates from the proximity effect in the normal metal. It is characterized by the anomalous Green function $\mathscr{F}(\mathbf{r}, \mathbf{r}') \equiv F(\mathbf{r}, \mathbf{r}', t = t')$, which is connected to $\Delta(\mathbf{r}, \mathbf{r}')$ by the interaction constant. Due to the lack of symmetry at the boundary of a puddle, an s-wave component $\mathscr{F}(\mathbf{r} = \mathbf{r}') = \mathscr{F}^{(S)}(\mathbf{r}) \neq 0$ of the anomalous Green function is generated in the neighboring metal. At a distance from the superconductor-normal metal boundary larger than the elastic electron mean free path the anomalous Green function becomes isotropic. In other words, only the s-component $\mathscr{F}(\mathbf{r} = \mathbf{r}') = \mathscr{F}^{(S)}(\mathbf{r})$ survives. It is this component that propagates between far separated puddles and determines the Joshepson coupling.

The sign of $\mathscr{F}^{(S)}(\mathbf{r})$ at a normal metal-superconductor boundary is determined by the sign of the d-wave order parameter in the $\mathbf{k}$-direction perpendicular to the boundary. Therefore, it changes along the boundary of a puddle.

At a distance from an individual i-th puddle larger than its size and smaller than the distance between the puddles the quantity $\mathscr{F}^{s}(\mathbf{r})$ has a sign $\eta_i = \pm 1$, which depends on the shape of the i-th puddle. This point is illustrated in Fig. 2a, where the sign of the anomalous Green function is positive in hatch-marked areas, and negative outside of these areas.

If the distance between puddles is larger than their size, the sign of the Joshepson coupling energy E_{Jos} is determined by a product $\eta_i \eta_j$

$$E_{Jos} = \sum_{i \neq j} \eta_i \eta_j J_{ij}^{(S)} \cos(\phi_i - \phi_j). \quad (1)$$

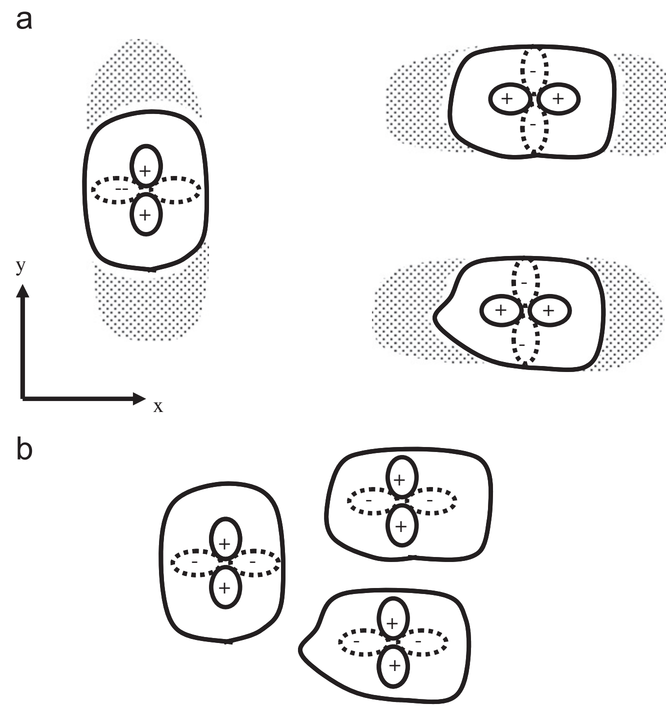

Fig. 2. A qualitative illustration of the global d-wave to s-wave transition. Solid lines represent boundaries of d-wave superconducting puddles embedded into a normal metal. Hatch-marked areas indicate the areas were the s-wave component of the anomalous Green function $F^s(\mathbf{r}, \mathbf{r})$ is positive. Outside these areas $F^s(\mathbf{r}, \mathbf{r})$ is negative. (a) The case of small puddle concentration when the system has s-wave global symmetry. (b) The case of big puddle concentration when the system has a global d-wave symmetry.

Here indexes i, j label puddles, $J_{ij}^{(S)} > 0$. Eq. (1) represents the Mattis model, which is well known in the theory of spin glasses [11]. The ground state of this model corresponds to

$$\cos(\phi_i) = -\eta_i. \quad (2)$$

Thus the distribution of $\cos(\phi_i)$ between puddles looks completely random as it is shown in Fig. 1b. However, the system is not a glass because its ground state has a hidden symmetry. In other words if the distances between puddles are bigger than the characteristic size of the puddles, R, the Josephson coupling between puddles inevitably favors globally s-wave superconductivity, even though the order parameter on each puddle looks locally d-wave-like. It is obvious that at a high concentration of puddles, the order parameter in the ground state has global d-wave symmetry. (See Fig. 2b.)

At intermediate distances, the situation is more complicated. Areas with different signs of $\mathscr{F}^{(S)}(\mathbf{r})$ mix in a random fashion. We argue that the most important aspects of this complex situation can be modeled by adding to the right hand side of Eq. (1) a term

$$\sum_{i \neq j} J_{ij}^{(d)} \cos(\phi_i - \phi_j), \quad (3)$$

where $J_{ij}^{(d)} > 0$ characterizes the strength of the exchange interaction between the d-wave components of the order parameter. Typically, at small $|\mathbf{r}_i - \mathbf{r}_j|$, $J_{ij}^{(d)} > J_{ij}^{(S)}$, but at large $|\mathbf{r}_i - \mathbf{r}_j|$ the coupling strength $J_{ij}^{(S)}$ decays more slowly than $J_{ij}^{(d)}$. Here $\mathbf{r}_i$ are coordinates of the puddles. Thus, it is likely that in this intermediate region the system may exhibit spin glass features and/or coexistence of d-wave and s-wave ordering. In this article, however, we will not further explore this fascinating but complex aspect of this problem.

To quantify the picture presented above one has to compute the Josephson coupling between a pair of far separated puddles.