



One-dimensional spin-1 ferromagnetic Heisenberg model with exchange anisotropy and single-ion anisotropy under external magnetic field

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ABSTRACT

The magnetic properties of the one-dimensional spin-1 ferromagnetic Heisenberg model are investigated by Green's function method. The magnetic properties of the system are treated by the random phase approximation for the exchange interaction term, and the Anderson–Callen approximation for the single-ion anisotropy term. The critical temperature, magnetization, and susceptibility are found to be dependent of the anisotropies. Our results are in agreement with the other theoretical results.

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1. Introduction

Recently, low dimensional magnetic properties of the ferromagnetic Heisenberg model have received considerable interest. There are many theoretical methods which have been employed to investigate the magnetic properties of the system, such as spin wave theory [1,2], mean-field theory [3], Monte Carlo method [4], renormalization group method [5,6], Green's function method [7–10], and so on.

From the theoretical standpoint, it is known that the spin wave theory, mean-field theory, and thermodynamic perturbation theory are only applied to the low temperature, critical temperature, and above critical temperature areas, respectively. The Monte Carlo method and renormalization group method are also good tools to study the ferromagnetic Heisenberg model, but they are too tedious. Whereas, Green's function is the better method, as it is simple and can be applied to all temperature areas. As are pointed out in Refs. [7,11], the two-time Green's function method is considered to be the standard method for magnetic systems. Using this method, we will obtain a nonlinear differential equation of motion. To get tractable solutions, the decoupling method has to be used, such as the random phase approximation

(RPA) [12,13], Callen approximation (CA) [14], and Anderson–Callen approximation (ACA) [15]. These approximations can be given good results, which agree with the theoretical and experimental results in a wide range of temperatures and magnetic fields [10,16,17].

As are pointed out in Refs. [12–14], RPA and CA both yield a good approximation to the Heisenberg ferromagnetic interaction, but RPA provides a simpler way to obtain the magnetic properties than CA. However, the term resulting from the single-ion anisotropy has to be treated differently. If RPA yields for the single-ion anisotropy, it will lead to unphysical results. Instead, ACA is shown to be a good approximation in this situation [15,16]. For two- and three-dimensional cases of single-ion anisotropy, ACA has been used to study field-induced magnetic reorientation [16–18].

In this paper, we apply the two-time Green's function method to investigate one-dimensional (1D) quantum Heisenberg ferromagnet with the exchange anisotropy and single-ion anisotropy. The 2D and 3D magnetic properties of this system have been known [16,18]. Our motivation to study this problem is less analytical study on 1D spin-1 magnetic properties affected by the magnetic field. Most theoretical studies were confined in the ferromagnetic chain only with the single-ion anisotropy [19–23], and failed to give the magnetic behavior dependent explicitly upon the exchange interaction and the anisotropic parameters.

In this work, the Hamiltonian of the model includes the anisotropic ferromagnetic interaction, external magnetic field,

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exchange anisotropy and single-ion anisotropy. The magnetic properties of the system are treated by RPA for the exchange interaction term and ACA for the anisotropy term. The critical temperature, magnetization and susceptibility are found to be as a function of the temperature, magnetic field and anisotropies. Our results are in agreement with the other theoretical results.

A brief outline of this paper is as follows. In Section 2, we give the Heisenberg model. And using the method of Green's function, we employ RPA and ACA to establish the self-consistent equations. In Section 3, we present our numerical results and investigate the effect of external magnetic field, exchange anisotropy and single-ion anisotropy on the magnetization and susceptibility. Finally, brief conclusions are given in Section 4.

2. Model

The 1D spin-1 ferromagnetic Heisenberg model with the exchange anisotropy and single-ion anisotropy under the external magnetic field can be described by the following Hamiltonian:

$$H = -\frac{J}{2} \sum_{\langle ij \rangle} [a(S_i^x S_j^x + S_i^y S_j^y) + S_i^z S_j^z] - b \sum_i (S_i^z)^2 - h \sum_i S_i^z, \quad (1)$$

where J is the exchange interaction ($J > 0$), and h is the external magnetic field. a and b denote the exchange anisotropy and single-ion anisotropy, respectively. The first summation runs over pairs of nearest-neighbor sites, and the second and third over all sites of lattice. S_i^x , S_i^y and S_i^z represent the three components of spin operator at site i . Eq. (1) is the Ising model and isotropic ferromagnet for $a = b = 0$ and $a = 1$, $b = 0$, respectively.

In the following, we apply the spin raising and lowering operators, $S_i^\pm = S_i^x \pm iS_i^y$, to simplify the above Hamiltonian, which can be rewritten as

$$H = -\frac{J}{2} \sum_{\langle ij \rangle} \left[\frac{a}{2} (S_i^+ S_j^- + S_i^- S_j^+) + S_i^z S_j^z \right] - b \sum_i (S_i^z)^2 - h \sum_i S_i^z. \quad (2)$$

In order to obtain the magnetic properties of this system, we utilize operators of S_i^+ and S_j^- to form Green's functions, i.e.

$$\langle \langle S_i^+(t); S_j^- \rangle \rangle = -i\theta(t) \langle S_i^+(t) S_j^- - S_j^- S_i^+(t) \rangle, \quad (3)$$

$$\langle \langle S_i^+(t); (S_j^-)^2 S_j^+ \rangle \rangle = -i\theta(t) \langle S_i^+(t) (S_j^-)^2 S_j^+ - (S_j^-)^2 S_j^+ S_i^+(t) \rangle. \quad (4)$$

The equation of motion for $S_i^+(t)$ can be written as

$$i \frac{d}{dt} S_i^+(t) = [S_i^+(t), H] = -J \sum_j [a S_i^z(t) S_j^+(t) - S_i^+(t) S_j^z(t)] + b [S_i^+(t) S_i^z(t) + S_i^z(t) S_i^+(t)] + h S_i^+(t). \quad (5)$$

Using Eq. (5), we can obtain the equations of motion for Green's functions

$$i \frac{d}{dt} \langle \langle S_i^+(t); S_j^- \rangle \rangle = 2\delta(t) \langle \langle S_i^z \rangle \rangle \delta_{ij} + h \langle \langle S_i^+(t); S_j^- \rangle \rangle - J \sum_l \langle \langle a S_i^z(t) S_l^+(t) - S_i^+(t) S_l^z(t); S_j^- \rangle \rangle + b \langle \langle S_i^+(t) S_i^z(t) + S_i^z(t) S_i^+(t); S_j^- \rangle \rangle, \quad (6)$$

$$i \frac{d}{dt} \langle \langle S_i^+(t); (S_j^-)^2 S_j^+ \rangle \rangle = \delta(t) \langle [S_i^+, (S_j^-)^2 S_j^+] \rangle + h \langle \langle S_i^+(t); (S_j^-)^2 S_j^+ \rangle \rangle - J \sum_l \langle \langle a S_i^z(t) S_l^+(t) - S_i^+(t) S_l^z(t); (S_j^-)^2 S_j^+ \rangle \rangle + b \langle \langle S_i^+(t) S_i^z(t) + S_i^z(t) S_i^+(t); (S_j^-)^2 S_j^+ \rangle \rangle. \quad (7)$$

In order to get a closed set of equations, the higher order Green's functions on the right hand side of the equations should be decoupled. As mentioned above, we apply the RPA [12] for the exchange coupling terms

$$\langle \langle S_i^z S_k^+; S_j^- \rangle \rangle = \langle S_i^z \rangle \langle \langle S_k^+; S_j^- \rangle \rangle, \quad (8)$$

and the ACA [15,16] for the single-ion coupling terms

$$\langle \langle S_i^+ S_i^z + S_i^z S_i^+; S_j^- \rangle \rangle = \left[2 \langle S_i^z \rangle - \frac{1}{2S^2} \langle S_i^z \rangle (\langle S_i^+ S_i^- \rangle + \langle S_i^- S_i^+ \rangle) \right] \times \langle \langle S_i^+; S_j^- \rangle \rangle. \quad (9)$$

For $S = 1$, using the following relationships:

$$\begin{cases} S_i^- S_i^+ = 2 - S_i^z - (S_i^z)^2, \\ S_i^+ S_i^- = 2 + S_i^z - (S_i^z)^2, \end{cases} \quad (10)$$

Eq. (9) can be rewritten to be

$$\langle \langle S_i^+ S_i^z + S_i^z S_i^+; S_j^- \rangle \rangle = \langle S_i^z \rangle \langle (S_i^z)^2 \rangle \langle \langle S_i^+; S_j^- \rangle \rangle. \quad (11)$$

Here the values of magnetization $\langle S_i^z \rangle$ and the z self-correlation function $\langle (S_i^z)^2 \rangle$ are considered to be in dependence of its sites i , and we set $m = \langle S^z \rangle$ and $\langle (S_i^z)^2 \rangle = \langle (S^z)^2 \rangle$ in the following.

With the help of the RPA (8) and ACA (11), the equations of motion for Green's functions (6) and (7) become

$$i \frac{d}{dt} \langle \langle S_i^+(t); S_j^- \rangle \rangle = 2m\delta(t)\delta_{ij} + h \langle \langle S_i^+(t); S_j^- \rangle \rangle + bm \langle (S^z)^2 \rangle \langle \langle S_i^+(t); S_j^- \rangle \rangle - Jm \sum_l [a \langle \langle S_l^+(t); S_j^- \rangle \rangle - \langle \langle S_l^+(t); S_j^- \rangle \rangle], \quad (12)$$

$$i \frac{d}{dt} \langle \langle S_i^+(t); (S_j^-)^2 S_j^+ \rangle \rangle = 2[2+m-3\langle (S^z)^2 \rangle] \delta(t)\delta_{ij} + h \langle \langle S_i^+(t); (S_j^-)^2 S_j^+ \rangle \rangle - Jm \sum_l [a \langle \langle S_l^+(t); (S_j^-)^2 S_j^+ \rangle \rangle - \langle \langle S_l^+(t); (S_j^-)^2 S_j^+ \rangle \rangle] + bm \langle (S^z)^2 \rangle \langle \langle S_i^+(t); (S_j^-)^2 S_j^+ \rangle \rangle. \quad (13)$$

Here the well-known relationships for $S = 1$,

$$(S^z)^3 = S^z, \quad (S^z)^4 = (S^z)^2 \quad (14)$$

have been used in Eq. (13).

After Fourier transforming these equations with respect to the space and time variable

$$\langle \langle S_i^+(t); S_j^- \rangle \rangle = \frac{1}{N} \sum_k \int_{-\infty}^{+\infty} \frac{dE}{2\pi} g_k(E) e^{i\vec{k} \cdot (\vec{i} - \vec{j}) - Et}, \quad (15)$$

$$\langle \langle S_i^+(t); (S_j^-)^2 S_j^+ \rangle \rangle = \frac{1}{N} \sum_k \int_{-\infty}^{+\infty} \frac{dE}{2\pi} f_k(E) e^{i\vec{k} \cdot (\vec{i} - \vec{j}) - Et}, \quad (16)$$

we can obtain the Fourier-transform solutions for Green's functions

$$g_k(E) = \frac{2m}{E - E(k)}, \quad (17)$$

$$f_k(E) = \frac{2[2+m-3\langle (S^z)^2 \rangle]}{E - E(k)}, \quad (18)$$

with

$$E(k) = 2Jm(1 - \cos k) + bm \langle (S^z)^2 \rangle + h. \quad (19)$$

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