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Plasmons and polaritons in a semi-infinite plasma and a plasma slab

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ABSTRACT

Plasmon and polariton modes are derived for an ideal semi-infinite (half-space) plasma and an ideal plasma slab by using a general, unifying procedure, based on equations of motion, Maxwell's equations and suitable boundary conditions. Known results are re-obtained in much a more direct manner and new ones are derived. The approach consists of representing the charge disturbances by a displacement field in the positions of the moving particles (electrons). The dielectric response and the electron energy loss are computed. The surface contribution to the energy loss exhibits an oscillatory behaviour in the transient regime near the surfaces. The propagation of an electromagnetic wave in these plasmas is treated by using the retarded electromagnetic potentials. The resulting integral equations are solved and the reflected and refracted waves are computed, as well as the reflection coefficient. For the slab we compute also the transmitted wave and the transmission coefficient. Generalized Fresnel's relations are thereby obtained for any incidence angle and polarization. Bulk and surface plasmon–polariton modes are identified. As it is well known, the field inside the plasma is either damped (evanescent) or propagating (transparency regime), and the reflection coefficient for a semi-infinite plasma exhibits an abrupt enhancement on passing from the propagating regime to the damped one (total reflection). Similarly, apart from characteristic oscillations, the reflection and transmission coefficients for a plasma slab exhibit an appreciable enhancement in the damped regime.

1. Introduction

After the discovery of bulk plasmons in an infinite electron plasma [1–3], there was a great deal of interest in plasmons occurring in structures with special geometries, like a half-space (semi-infinite) plasma, a plasma slab of finite thickness, a two-plasmas interface (two plasmas bounding each other), a two-dimensional sheet with an aperture, a slab with a cylindrical hole, structures with surface gratings or regular holes patterns, layered films, cylindrical rods and spherical particles, etc. There is a vast literature on various structures with special geometries exhibiting plasmon modes. These studies were aimed mainly at identifying new plasmon modes, like the surface plasmons [4–11], accounting for the electron energy loss experiments and exploring the interaction of the electron plasma with electromagnetic radiation (polariton excitations) [12–24]. More recently, a possible enhancement of the electromagnetic radiation scattered on electron plasmas with special geometries enjoyed a particular interest [25–27]. In all these studies the plasmon and polariton modes are of fundamental importance [28–32]. The methods used in deriving such results are of great diversity, resorting often to particular assumptions, such that the basic underlying mechanism of plasmons or polaritons' occurrence is often obscured. The need is therefore felt of having a general, unifying procedure for deriving plasmon and polariton modes in structures with special geometries, as based on the equation of motion of the charge density, Maxwell's equations and the corresponding boundary conditions. Such a procedure is presented in this paper for an ideal semi-infinite plasma and an ideal plasma slab.

We represent the charge disturbances as $\delta n = -n \operatorname{div} \mathbf{u}$, where n is the (constant, uniform) charge concentration and $\mathbf{u}$ is a displacement field of the mobile charges (electrons). This representation is valid for $\mathbf{K}\mathbf{u}(\mathbf{K}) \ll 1$, where $\mathbf{K}$ is the wavevector and $\mathbf{u}(\mathbf{K})$ is the Fourier component of the displacement field. We assume a rigid neutralizing background of positive charge, as in the well-known jellium model. In the static limit, i.e. for Coulomb interaction, the Lagrangian of the electrons can be written as

$$L = \int d\mathbf{r}\left[\frac{1}{2}mn\dot{\mathbf{u}}^2 - \frac{1}{2}\int d\mathbf{r}' U(|\mathbf{r}-\mathbf{r}'|)\delta n(\mathbf{r})\delta n(\mathbf{r}')\right] + e\int d\mathbf{r}\Phi(\mathbf{r})\delta n(\mathbf{r}), \quad (1)$$

where m is the electron mass, $U(r) = e^2/r$ the Coulomb energy, $-e$ the electron charge and $\Phi(\mathbf{r})$ the external scalar potential. Eq. (1)

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leads to the equation of motion

$$m\ddot{\mathbf{u}} = n\,\mathrm{grad}\int d\mathbf{r}' U(|\mathbf{r}-\mathbf{r}'|)\,\mathrm{div}\,\mathbf{u}(\mathbf{r}') + e\,\mathrm{grad}\,\Phi \tag{2}$$

which is the starting equation of our approach. We leave aside the dissipation effects (which can easily be included in Eq. (2)).

By using the Fourier transform for an infinite plasma it is easy to see that the eigenmode of the homogeneous Eq. (2) is the well-known bulk plasmon mode given by $\omega_p^2 = 4\pi ne^2/m$. On the other side, equation $\delta n = -n\,\mathrm{div}\,\mathbf{u}$ is equivalent with Maxwell's equation $\mathrm{div}\,\mathbf{E}_i = -4\pi e\delta n$, where $\mathbf{E}_i = 4\pi ne\mathbf{u}$ is the internal electric field (equal to $-4\pi\mathbf{P}$, where $\mathbf{P}$ is the polarization). Making use of the electric displacement $\mathbf{D} = -\mathrm{grad}\,\Phi = \varepsilon(\mathbf{D}+\mathbf{E}_i)$, we get the well-known dielectric function $\varepsilon = 1-\omega_p^2/\omega^2$ in the long-wavelength limit from the solution of the inhomogeneous Eq. (2). Similarly, since the current density is $\mathbf{j} = -en\dot{\mathbf{u}}$, we get the well-known electrical conductivity $\sigma = i\omega_p^2/4\pi\omega$.

We apply this approach to a semi-infinite plasma and a plasma slab. First, we derive the surface and bulk plasmon modes and obtain the dielectric response and the electron energy loss for a semi-infinite plasma. The surface contribution to the energy loss exhibits an oscillatory behaviour in the transient regime near the surface. Further on, we consider the interaction of the semi-infinite plasma with the electromagnetic field, as described by the usual term $(1/c)\int d\mathbf{r}\mathbf{j}\mathbf{A} - \int d\mathbf{r}\rho\Phi$ in the Lagrangian, where $\mathbf{A}$ is the vector potential, $\rho = en\,\mathrm{div}\,\mathbf{u}$ is the charge density and Φ is the scalar potential. We limit ourselves to the interaction with the electric field, and compute the reflected and refracted waves, as well as the reflection coefficient. Generalized Fresnel's relations are obtained for any incidence angle and polarization. We find it more convenient to use the radiation formulae for the retarded potentials, instead of using directly the Maxwell's equations, and the resulting integral equations are solved. Bulk and surface plasmon–polariton modes are identified. The field inside the plasma is either damped (evanescent) or propagating (transparency regime), and the reflection coefficient exhibits an abrupt enhancement on passing from the propagating to the damping regime (total reflection). Finally, we give similar results for a plasma slab, where we compute also the transmitted field and the transmission coefficient. Apart from characteristic oscillations, the reflection and transmission coefficients for a plasma slab exhibit an appreciable enhancement in the damped regime. The present approach can be extended to various other plasma structures with special geometries.

2. Plasma eigenmodes

We consider an ideal semi-infinite plasma extending over the half-space $z>0$ (and bounded by the vacuum for $z<0$). The displacement field $\mathbf{u}$ is then represented as $(\mathbf{v}, u_3)\theta(z)$, where $\mathbf{v}$ is the displacement component in the (x,y)-plane, u_3 is the displacement component along the z-direction and $\theta(z)=1$ for $z>0$ and $\theta(z)=0$ for $z<0$ is the step function. In equation of motion (2) $\mathrm{div}\,\mathbf{u}$ is then replaced by

$$\mathrm{div}\,\mathbf{u} = \left(\mathrm{div}\,\mathbf{v} + \frac{\partial u_3}{\partial z}\right)\theta(z) + u_3(0)\delta(z), \tag{3}$$

where $u_3(0) = u_3(\mathbf{r}, z=0)$, $\mathbf{r}$ being the in-plane (x,y) position vector. Eq. (2) becomes

$$m\ddot{\mathbf{u}} = ne^2\,\mathrm{grad}\int d\mathbf{r}'dz' \frac{1}{\sqrt{(\mathbf{r}-\mathbf{r}')^2+(z-z')^2}} \times\left[\mathrm{div}\,\mathbf{v}(\mathbf{r}'.z') + \frac{\partial u_3(\mathbf{r}',z')}{\partial z'}\right]$$
$$+ne^2\,\mathrm{grad}\int d\mathbf{r}' \frac{1}{\sqrt{(\mathbf{r}-\mathbf{r}')^2+z^2}} u_3(\mathbf{r}',0) + e\,\mathrm{grad}\,\Phi \tag{4}$$

for $z>0$. One can see the (de)-polarizing field occurring at the free surface $z=0$ (the second integral in Eq. (4)).

We use Fourier transforms of the type

$$\mathbf{u}(r,z;t) = \sum_{\mathbf{k}}\int d\omega\,\mathbf{u}(\mathbf{k},z;\omega)e^{i\mathbf{k}\mathbf{r}}e^{-i\omega t} \tag{5}$$

(for in-plane unit area), as well as the Fourier representation

$$\frac{1}{\sqrt{r^2+z^2}} = \sum_{\mathbf{k}}\frac{2\pi}{k}e^{-k|z|}e^{i\mathbf{k}\mathbf{r}} \tag{6}$$

for the Coulomb potential. Then, it is easy to see that Eq. (4) leads to the integral equation

$$\omega^2 v = \frac{1}{2}k\omega_p^2\int_0^\infty dz' v e^{-k|z-z'|} + \frac{1}{2k}\omega_p^2\int_0^\infty dz'\frac{\partial v}{\partial z'}\frac{\partial}{\partial z'}e^{-k|z-z'|} - \frac{iek}{m}\Phi \tag{7}$$

and $iku_3 = \partial v/\partial z$, where we have dropped out for simplicity the arguments $\mathbf{k}, z$ and ω. The $\mathbf{v}$-component of the displacement field is directed along the wavevector $\mathbf{k}$ (in-plane longitudinal waves). This integral equation can easily be solved. Integrating by parts in its *rhs* we get

$$\omega^2 v = \omega_p^2 v - \frac{1}{2}\omega_p^2 v_0 e^{-kz} - \frac{iek}{m}\Phi, \tag{8}$$

hence

$$v = \frac{iek\omega_p^2}{m}\frac{\Phi_0}{(\omega^2-\omega_p^2)(2\omega^2-\omega_p^2)}e^{-kz} - \frac{iek}{m}\frac{\Phi}{\omega^2-\omega_p^2}$$
$$u_3 = -\frac{ek\omega_p^2}{m}\frac{\Phi_0}{(\omega^2-\omega_p^2)(2\omega^2-\omega_p^2)}e^{-kz} - \frac{e}{m}\frac{\Phi'}{\omega^2-\omega_p^2}, \tag{9}$$

where $v_0 = v(z=0)$, $\Phi_0 = \Phi(z=0)$ and $\Phi' = \partial\Phi/\partial z$. One can see the surface contributions (terms proportional to $\Phi_0 e^{-kz}$) and bulk contributions (Φ, Φ'-terms).

The solutions given by Eqs. (9) exhibit two eigenmodes, the bulk plasmon $\omega_b = \omega_p$ and the surface plasmon $\omega_s = \omega_p/\sqrt{2}$, as it is well known. Indeed, the homogeneous Eq. (8) ($\Phi = 0$) has two solutions: the surface plasmon $v = v_0 e^{-kz}$ for $\omega^2 = \omega_p^2/2$ and the bulk plasmon $v_0 = 0$ for $\omega^2 = \omega_p^2$. Making use of this observation we can represent the general solution as an eigenmodes series

$$v(\mathbf{k},z) = \sqrt{2k}v_0(\mathbf{k})e^{-kz} + \sum_\kappa\sqrt{\frac{2k^2}{\kappa^2+k^2}}v(\mathbf{k},\kappa)\sin\kappa z \tag{10}$$

for $z>0$, where $v(\mathbf{k},-\kappa) = -v(\mathbf{k},\kappa)$ and $iku_3(\mathbf{k},z) = \partial v(\mathbf{k},z)/\partial z$. Then, it is easy to see that the hamiltonian $H = T+U$ corresponding to the Lagrangian $L = T-U$ given by Eq. (1) becomes

$$T = nm\sum_{\mathbf{k}}\dot{v}_0^*(\mathbf{k})\dot{v}_0(\mathbf{k}) + nm\sum_{\mathbf{k}\kappa}\dot{v}^*(\mathbf{k},\kappa)\dot{v}(\mathbf{k},\kappa)$$
$$U = 2\pi n^2e^2\sum_{\mathbf{k}}v_0^*(\mathbf{k})v_0(\mathbf{k}) + 4\pi n^2e^2\sum_{\mathbf{k}\kappa}v^*(\mathbf{k},\kappa)v(\mathbf{k},\kappa), \tag{11}$$

where T is the kinetic energy and U is the potential energy. We can see that this hamiltonian corresponds to harmonic oscillators with frequencies $\omega_s = \omega_p/\sqrt{2}$ and $\omega_b = \omega_p$.

Making use of $\mathbf{E}_i = 4\pi ne\mathbf{u}$ and Eqs. (9) we can write down the internal field (polarization) as

$$E_\perp(\mathbf{k},z;\omega) = \frac{ik\omega_p^4\Phi(\mathbf{k},0;\omega)}{(\omega^2-\omega_p^2)(2\omega^2-\omega_p^2)}e^{-kz} - \frac{ik\omega_p^2\Phi(\mathbf{k},z;\omega)}{\omega^2-\omega_p^2}$$
$$E_\parallel(\mathbf{k},z;\omega) = -\frac{k\omega_p^4\Phi(\mathbf{k},0;\omega)}{(\omega^2-\omega_p^2)(2\omega^2-\omega_p^2)}e^{-kz} - \frac{\omega_p^2\Phi'(\mathbf{k},z;\omega)}{\omega^2-\omega_p^2}, \tag{12}$$