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Crossover from non-Fermi liquid to Fermi liquid behavior close to a quantum critical point: A brief review

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ABSTRACT

The nesting of the Fermi surfaces of an electron and a hole pocket separated by a nesting vector **Q** and the interaction between electrons gives rise to itinerant antiferromagnetism. The order can gradually be suppressed by mismatching the nesting and a quantum critical point is obtained as the Néel temperature tends to zero. We review our results on the specific heat, the quasi-particle linewidth, the electrical resistivity, the amplitudes of de Haas–van Alphen oscillations and the dynamical spin susceptibility.

1. Introduction

Landau's Fermi liquid (FL) theory has been successful in describing the low energy properties of most normal metals. Numerous U, Ce and Yb based heavy fermion systems [1–3] display deviations from FL behavior, which manifest themselves as, e.g., a $\log(T)$-dependence in the specific heat over T, C/T, a singular behavior at low T of the magnetic susceptibility, χ, and a power-law dependence of the resistivity, ρ, with an exponent close to one. These deviations from FL are known as non-Fermi liquid (NFL) behavior. The breakdown of the FL can be tuned by alloying (chemical pressure), hydrostatic pressure or the magnetic field. In most cases the systems are close to the onset of antiferromagnetism (AF) and the NFL behavior is attributed to a quantum critical point (QCP) [4–12].

Recently we studied the pre-critical region of a heavy electron band with two parabolic pockets, one electron-like and the other hole-like, separated by a wave vector **Q** using (i) a field-theoretical multiplicative renormalization group (RG) approach [9] and (ii) the Wilsonian RG that eliminate the fast degrees of freedom close to an ultraviolet cutoff and rewrite the Hamiltonian in terms of renormalized slow variables [12]. The interaction is the remaining repulsion between heavy quasi-particles after the heavy particles have been formed in the sense of a Fermi liquid and is assumed to be weak. The interaction between the electrons induces itinerant AF or charge density waves (CDW) due to the nesting of the Fermi surfaces of the two pockets. For perfect nesting (electron–hole symmetry) an arbitrarily small interaction is sufficient for a ground state with long-range order. The degree of nesting is controlled by the mismatch parameter, $\delta = \frac{1}{2}|k_{F1} - k_{F2}|v_F$ [k_{F1} (k_{F2}) is the Fermi momentum of the electron (hole) pocket]. In this way the ordering temperature can be tuned to zero, leading to a QCP.

In this paper we review our main results. In the paramagnetic phase the effective mass, m^* (specific heat over T, C/T) and the magnetic susceptibility increase logarithmically as T is lowered and diverge at the critical point signaling the breakdown of the FL [9,12]. There is a crossover from the $-\ln(T)$ dependence of C/T to constant γ as T is lowered if the QCP is not perfectly tuned, in agreement with experiments on numerous systems. The quasi-particle linewidth shows a crossover from NFL ($\sim T$) to FL ($\sim T^2$) behavior with increasing nesting mismatch and decreasing temperature [13]. The electrical resistivity [14], the dynamical susceptibility [15] and the amplitudes of the de Haas–van Alphen oscillations [16] have also been studied.

The response function to superconductivity diverges as T_N is approached [17], but the dominating correlations are AF. NFL behavior, AF order and superconductivity in the neighborhood of a QCP have been observed in $CePd_2Si_2$ and $CeIn_3$ under pressure [18]. We have also investigated the renormalization of the electron–phonon coupling, the softening of the phonon with wave vector **Q** and the consequences of this softening on the thermal expansion [19].

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2. Two-pocket model

The model consists of two pockets, one electron-like and the other one hole-like, separated by a wavevector $\mathbf{Q}$. The kinetic energy of the carriers is given by [9,12]

$$H_0 = \sum_{\mathbf{k}\sigma}\left[\varepsilon_1(\mathbf{k})c^\dagger_{1\mathbf{k}\sigma}c_{1\mathbf{k}\sigma} + \varepsilon_2(\mathbf{k})c^\dagger_{2\mathbf{k}\sigma}c_{2\mathbf{k}\sigma}\right], \quad (1)$$

where $\mathbf{k}$ is measured from the center of each pocket, and assumed to be small compared to the nesting vector $\mathbf{Q}$. Here $\varepsilon_1(\mathbf{k}) = v_F(k - k_{F1})$ and $\varepsilon_2(\mathbf{k}) = v_F(k_{F2} - k)$, and for simplicity we assume that the Fermi velocity is the same for both pockets.

A strong interaction between electrons gives rise to heavy fermion bands. In the spirit of the FL theory, there are weak remaining interactions between the heavy quasi-particles after the heavy particles are formed. The heavy electron bands are described by Eq. (1) and the weak interactions between quasi-particles are given by [9,12]

$$H_{12} = V\sum_{\mathbf{k}\mathbf{k}'\mathbf{q}\sigma\sigma'} c^\dagger_{1\mathbf{k}+\mathbf{q}\sigma}c_{1\mathbf{k}\sigma}c^\dagger_{2\mathbf{k}'-\mathbf{q}\sigma'}c_{2\mathbf{k}'\sigma'} + U\sum_{\mathbf{k}\mathbf{k}'\mathbf{q}\sigma\sigma'} c^\dagger_{1\mathbf{k}+\mathbf{q}\sigma}c^\dagger_{2\mathbf{k}'-\mathbf{q}\sigma'}c_{1\mathbf{k}\sigma'}c_{2\mathbf{k}'\sigma}. \quad (2)$$

Here V and U represent the interaction strength for small ($|\mathbf{q}| \ll |\mathbf{Q}|$) and large (of the order of $\mathbf{Q}$) momentum transfer between the pockets, respectively. The limit of the Hubbard model is obtained by choosing $V = U$.

The leading order corrections to the vertex are the bubble diagrams of the zero-sound type (antiparallel propagator lines), which are logarithmic in the external energy ω. Assuming that ω is small compared to the cutoff energy D, and that the density of states for electrons and holes is constant, ρ_F, we have

$$\tilde{V} = \frac{V}{1-\rho_F V\xi}, \quad 2\tilde{U} - \tilde{V} = \frac{(2U-V)}{1+\rho_F(2U-V)\xi}, \quad (3)$$

where $\xi = \ln[D/(|\omega| + 2T + \delta)]$ [12]. A divergent vertex indicates strong coupling and signals an instability [9,12].

Within the logarithmic approximation the linear response to a staggered magnetic field, $\chi_S(\mathbf{Q},\omega)$, and to a CDW, $\chi_c(\mathbf{Q},\omega)$, are given by [9]

$$\chi_S(\mathbf{Q},\omega) = 2\xi\rho_F\tilde{V}/V,$$
$$\chi_c(\mathbf{Q},\omega) = 2\xi\rho_F(2\tilde{U}-\tilde{V})/(2U-V). \quad (4)$$

Hence, if $V>0$ a spin density wave is possible with a Néel temperature $T_N = \frac{1}{2}D\exp[-(\rho_F V)^{-1}] - \frac{1}{2}\delta$, and if $2U<V$ a CDW can be formed at $T_c = \frac{1}{2}D\exp\{-[\rho_F(V-2U)]^{-1}\} - \frac{1}{2}\delta$. The condition for a QCP is $T_N = 0$ or $T_c = 0$, and if $T_N<0$ and $T_c<0$ long-range order has not developed. Thus, for sufficiently large Fermi surface mismatch the renormalization does not lead to an instability [12]. The QCP is an unstable fixed point and can only be reached by perfectly tuning the system [9].

In the disordered phase the γ-coefficient of the specific heat is given by the effective thermal mass [9,12]

$$\frac{m^*(T)}{m} = \frac{\gamma}{\gamma_0} = 1 + \frac{\xi\rho_F^2}{4}[3V\tilde{V} + (2U-V)(2\tilde{U}-\tilde{V})], \quad (5)$$

where γ_0 refers to the non-interacting system. Here we kept only the leading logarithmic contributions, and ξ is to be taken with $\omega = 0$.

The T-dependence of C/T as a function of $\ln(T)$ is shown in Fig. 1. Here $\delta_0 = 0.07$ corresponds approximately to the critical mismatch. For the tuned QCP, C/T increases logarithmically as T is lowered and diverges at the critical point signaling the breakdown of the Fermi liquid [9,12]. If $\delta > \delta_0$ there is a crossover from the logarithmic dependence (NFL) to a constant C/T (FL) as T is lowered [13].

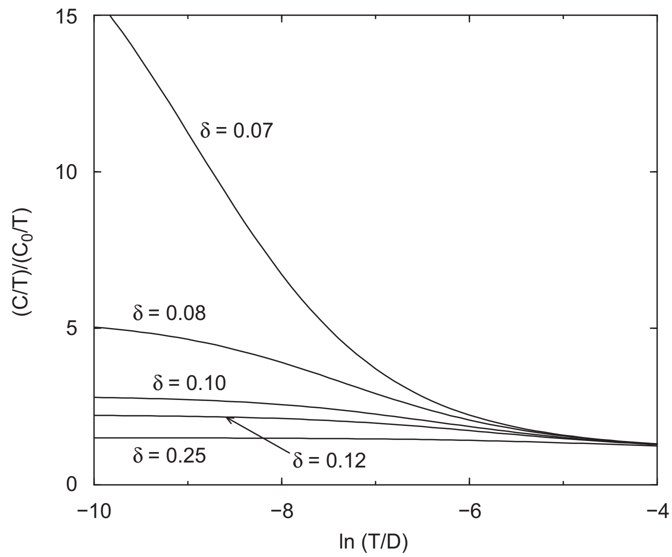

Fig. 1. Enhancement of the thermal mass as a function of $\ln(T)$ for $V\rho_F = U\rho_F = 0.2$, $D = 10$, and several mismatch parameters δ. $\delta_0 \approx 0.07$ is approximately the critical mismatch. Note the crossover from NFL to FL for $\delta > \delta_0$ as T is lowered.

3. Quasi-particle linewidth

In an FL the damping of the quasi-particles is proportional to T^2, while the nesting condition changes this behavior to a quasi-linear dependence in T. The linewidth Γ is calculated following a procedure outlined by Virosztek and Ruvalds [20] in the context of high-T_c superconductivity. In the disordered phase Γ is given by the imaginary part of the electron self-energy, which can be expressed as a convolution of a staggered susceptibility $\chi''_S(\omega/2T)$ with a fermion Green's function [13],

$$\Gamma_{NFL}(\omega,T) = \frac{1}{2}T\int dx\left[\coth(x) - \tanh\left(x - \frac{\omega}{2T}\right)\right] \times \chi''_S(x)[3|\tilde{V}|^2 + |2\tilde{U} - \tilde{V}|^2]\rho_F, \quad (6)$$

$$\chi''_S(\omega/2T) \approx \frac{\rho_F}{2}\mathrm{Im}\,\psi\left(\frac{1}{2} + \frac{\Gamma_{NFL}}{2\pi T} + i\frac{\omega - 2(\delta-\delta_0)}{4\pi T}\right) + \frac{\rho_F}{2}\mathrm{Im}\,\psi\left(\frac{1}{2} + \frac{\Gamma_{NFL}}{2\pi T} + i\frac{\omega + 2(\delta-\delta_0)}{4\pi T}\right), \quad (7)$$

where $\mathrm{Im}\,\psi$ is the imaginary part of the digamma function, ω is the external frequency, and δ_0 is the nesting mismatch corresponding to the QCP. The frequency in the vertices is $2T|x| + |\omega|/2$ and we use the analytic continuation of the vertex functions, i.e. $i\pi/2$ is added to ξ. The frequency of Γ_{NFL} in $\mathrm{Im}\,\psi$ is $2T|x|$. The self-consistent solution of Eqs. (6) and (7) yields the quasi-particle NFL linewidth as a function of ω and T [13].

There is also a FL contribution to the quasi-particle linewidth given by [13]

$$\Gamma_{FL}(\omega,T) = \frac{\pi}{8}[\omega^2 + (\pi T)^2][3V^2 + (2U - V^2)]\rho_F^3, \quad (8)$$

which is added to F_{NFL} assuming that Matthiessen's rule is valid. The vertices in Γ_{FL} are not dressed, since this contribution does not arise from the nesting condition.

The ω and T dependence of the self-consistent Γ_{NFL} can be understood from some limiting cases [13]. First, consider the perfectly tuned QCP, i.e. $\delta = \delta_0$, set $\omega = 0$ and neglect Γ_{NFL} in the