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Spin wave relaxation and magnetic properties in [M/Cu] super-lattices; M = Fe, Co and Ni

A. Fahmi, A. Qachaou *

Department de Physique, Laboratoire de Physique de la matière condensée, Faculté des sciences, B.P133-14 000 Kenitra, Morocco

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ABSTRACT

In this work, we study the elementary excitations and magnetic properties of the [M/Cu] super-lattices with: M = Fe, Co and Ni, represented by a Heisenberg ferromagnetic system with N atomic planes. The nearest neighbour (NN), next nearest neighbour (NNN) exchange, dipolar interactions and surface anisotropy effects are taken into account and the Hamiltonian is studied in the framework of the linear spin wave theory. In the presence of the exchange alone, the excitation spectrum $E(k)$ and the magnetization $\langle S^z \rangle / S$ analytical expressions are obtained using the Green's function formalism. The obtained relaxation time of the magnon populations is nearly the same in the Fe and Co-based super-lattices, while these magnetic excitations would last much longer in the Ni-based super lattice. A numerical study of the surface anisotropy and long-ranged dipolar interaction combined effects are also reported. The exchange integral values deduced from a comparison with experience for the three super-lattices are coherent.

1. Introduction

Recently, much interest has been given to the study of the thin magnetic films and multi-layers. Their new properties allow a wide applications in certain technological fields such as micro-electronics and high-density magneto-optics recording [1–3]. At fundamental level, several theoretical and experimental studies were performed to understand their magnetic behaviour. In the presence of the only short-ranged exchange interaction between nearest neighbours (NN) and next nearest neighbours (NNN) of pairs of spins, several works show that taking into account of the anisotropy can introduce a gap in the spin wave spectrum and thus lead to a non-divergent value for the magnetization at finite temperature [4–6]. These last works have used a classical continuum approach which gives good results for thick films. For ultra-thin films, the linear spin wave theory may give coherent results with experience if one takes into account of both dipolar interaction and surface anisotropy [7]. One of the most attractive features, both from fundamental physics and potential applications, is the combined effect of a long-ranged dipolar interaction and surface anisotropy which are always present in real systems. This combined effect has been studied by a number of authors in two-dimensional system [8] and in layered ferromagnetic film [9–11].

The purpose of this work is to study, using the spin wave theory, the properties of a super-lattice represented by a ferromagnetic Heisenberg model of localized spin with nearest neighbour and next nearest neighbour exchange interaction (J_1 and J_2). We admitted that each ultrathin ferromagnetic monolayer constitutes a quasi-two-dimensional system extending on a thickness of some atomic distances, although all its atoms are not necessarily located in the same plane. Thus, each super-lattice would consist of a succession of N ferromagnetic planes. We also took account of the surface anisotropy which plays an important role in the magnetic stability of this kind of system as proven by former experimental studies [12–14]. The effect of the dipolar interactions can be as significant as that of the anisotropy, the magnetic properties of the films result from a competition between the effects of long-ranged order dipolar interaction (D) and surface anisotropy (α). Indeed, α tends to align the moments perpendicular to the film plane whereas D favours their alignment in the plane. In paragraph 4, we reported a numerical processing of their influence on magnetization per spin. In absence of these two effects ($D = \alpha = 0$), we obtained an exact resolution for $N = 3$, 5 and 10 planes which we present in paragraph 3.

The comparison between the calculated and measured magnetization under magnetic field H allowed us to obtain a satisfactory estimation of the various exchange integral values.

2. The spin Hamiltonian

To describe our system, we suppose that the [M/Cu] super-lattices constitute a ferromagnetic system with localized spin.

* Corresponding author. Tel./fax: +212 537 37 27 70.
E-mail address: ahqachaou@yahoo.fr (A. Qachaou).

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The ferromagnetic film plane is taken parallel to the (*xz*) plane and then the *y* axis constitutes the normal to the film plane. The corresponding Hamiltonian including surface anisotropy, exchange and dipolar interaction terms can be expressed by

$$H = -J_1 \sum_{\langle ij \rangle} (S_i^x S_j^x + S_i^y S_j^y + \delta S_i^z S_j^z) - J_2 \sum_{\langle ij' \rangle} (\vec{S}_i \vec{S}_{j'}) - J_\perp \sum_{\langle ii' \rangle} (\vec{S}_i \vec{S}_{i'}) - \alpha \sum_i (S_i^y)^2 - g\mu_B H \sum_i S_i^z + \frac{g^2\mu_B^2}{2} \sum_{\langle ij \rangle} \frac{1}{r_{ij}^3} \left\{ \vec{S}_i \vec{S}_j - 3 \frac{(\vec{S}_i \vec{r}_{ij})(\vec{S}_j \vec{r}_{ij})}{r_{ij}^2} \right\} \quad (1)$$

J_1 and $J_2 > 0$ are, respectively, the NN and NNN exchange integrals in the same atomic plane. $J_\perp$ indicates the NN exchange integral belonging to two successive planes. $\delta > 0$ and $\alpha > 0$ represent the surface magneto crystalline anisotropies in plan and out of plan respectively. α is supposed acting only on the surface spins (planes 1 and *N*) and whose effect on the system properties is now well established. The external magnetic field applied along *z* axis contributes by a term $g\mu_B H$. The last term corresponds to the dipolar interaction between NN in the same plane and out of plane ($r_{ij} = |r_i - r_j|$).

The treatment of the Hamiltonian (1) within the framework of the linear spin wave theory consists in transforming the spin operators into boson creation a_{kl}^+ and annihilation a_{kl} operators according to the usual Holtstein–Primakoff transformation [15]. The Fourier transformation of these boson operators is introduced then to take account of the translation symmetry in the (*xz*) plane and of the break of this symmetry along the *y* axis. So the transformed Hamiltonian is

$$H = \sum_{l,m}^{N} \sum_{k_\parallel} \left\{ A_{lm}(k) a_{k_\parallel,l}^+ a_{k_\parallel,m} + \frac{1}{2} B_{lm} (a_{k_\parallel,l}^+ a_{-k_\parallel,m}^+ + a_{k_\parallel,l} a_{-k_\parallel,m}) \right\} \quad (2)$$

where the $A_{lm}(k)$ and $B_{lm}(k)$ terms are defined by

$$\begin{aligned} A_{lm}(k_\parallel) = {} & S \sum_{\gamma_\parallel} \left\{ 2J_1(\delta - \cos(k_\parallel \gamma_\parallel)) + D_{\gamma_\parallel} \left[\left(1 - \tfrac{3}{2} \left(\tfrac{r_{\gamma_\parallel}^x}{r_{\gamma_{|||}}} \right)^2 \right) \right. \right. \\ & \left. \left. \times \cos(k_\parallel \gamma_\parallel) - \left(1 - 3 \left(\tfrac{r_{\gamma_\parallel}^x}{r_{\gamma_\parallel}} \right)^2 \right) \right] \right\} \delta_{l,m} \\ & + S \sum_{\gamma'} \left\{ 2J_2(1 - \cos(k_\parallel \gamma')) + D_{\gamma_\parallel} \left[\left(1 - \tfrac{3}{2} \left(\tfrac{r_{\gamma'}^x}{r_{\gamma'}} \right)^2 \right) \right. \right. \\ & \left. \left. \times \cos(k_\parallel \gamma') - \left(1 - 3 \left(\tfrac{r_{\gamma'}^x}{r_{\gamma'}} \right)^2 \right) \right] \right\} \delta_{l,m} \\ & + S \sum_{\gamma_\perp} \left\{ \left[2J_\perp + D_{\gamma_\perp} \left(3 \left(\tfrac{r_{\gamma_\perp}^x}{r_{\gamma_\perp}} \right)^2 - 1 \right) \right] \right. \\ & \left. \times (2 - \delta_{l,1} - \delta_{l,N}) + h - \alpha(\delta_{l,1} + \delta_{l,N}) \right\} \delta_{l,m} \\ & - S \sum_{\gamma_\perp} \left[2J_\perp \cos(k_\parallel \gamma_\perp) - D_{\gamma_\perp} \left(1 - \tfrac{3}{2} \left(\tfrac{r_{\gamma_\perp}^x}{r_{\gamma_\perp}} \right)^2 \right) \cos(k_\parallel \gamma_\perp) \right] \\ & \times (\delta_{l,m-1} + \delta_{l,m+1}) \end{aligned} \quad (3a)$$

$$\begin{aligned} B_{lm}(k_{||}) = S & \left\{ -\frac{3}{2} \left[\sum_{\gamma_\parallel} D_{\gamma_\parallel} \left(\frac{r_{\gamma_{|||}}^x}{r_{\gamma_{|||}}} \right)^2 \cos(k_{|||} \gamma_\parallel) \right. \right. \\ & \left. \left. + \sum_{\gamma_\parallel} \left(\frac{r_{\gamma'}^x}{r_{\gamma'}} \right)^2 D_{\gamma'} \cos(k_\parallel \gamma') + \alpha(\delta_{l,1} + \delta_{l,N}) \right] \right\} \delta_{l,m} \\ & - \frac{3}{2} S \sum_{\gamma_\perp} D_{\gamma_\perp} \left(\frac{r_{\gamma_\perp}^x}{r_{\gamma_\perp}} \right)^2 \cos(k_\parallel \gamma_\parallel)(\delta_{l,m-1} + \delta_{l,m+1}) \end{aligned} \quad (3b)$$

$\gamma_{||}$, $\gamma_\perp$ and γ' present, respectively, the NN in plane, NN out plane and NNN in plane. $D_\gamma = g^2\mu_B^2/(r_\gamma)^3$ is the dipolar interaction constant, a is the lattice parameter, $h = g\mu_B H$ and $k = (k_x, k_z)$. *S* is the spin, δ_{lm} are elements of matrix identity, $k_\parallel$ and $k_\perp$ are the composantes of the wave vector in plane and out of plane, respectively.

The diagonalization of the Hamiltonian expressed by (2) and consequently the determination of the excitation spectrum as well as the magnetization per spin were carried out by using the Green's function method [8].

In the presence of both dipolar interactions and surface anisotropy ($D \neq 0$ and $\alpha \neq 0$), the analytical diagonalization of (2) becomes more complex, so, a numerical study is done in this case.

3. Analytical resolution: exchange effect only

For systems engaging transition metals, the exchange interactions are more important than surface anisotropy and dipolar interactions ($J_i \gg \alpha$ and $J_i \gg D$). Thus, an exact resolution can be made omitting the *D* and α contributions ($D = \alpha = 0$) in the Hamiltonian expressions (2) and (3). The magnetic field ($h = g\mu_B H$) has for simple effect to move the mode frequencies. The boson Hamiltonian is reduced thus to

$$\mathscr{H} = \sum_{lm} \sum_{k_{|||}} A_{lm}(k_\parallel) a_{k_\parallel,l}^+ a_{k_\parallel,m} \quad (4)$$

where $A_{lm}(k)$ is given by

$$A_{lm}(k) = 2S[J_1 \sum_{\gamma_{||}} (\Delta - e^{ik_{|||}\gamma_{|||}}) + J_2 \sum_{\gamma'} (1 - e^{ik_{|||}\gamma'}) + J_\perp] \delta_{lm} + 2J_\perp S \delta_{l,m+1}$$

3.1. Excitation spectra

The excitation spectra of the spin wave *E*(*k*) are obtained by solving the system of equations corresponding to the secular equation of the studied super lattices according to

$$\det(\underline{A} - E\underline{I}) = 0 \quad (5)$$

where ($\underline{A} - E\underline{I}$) is a matrix which is tridiagonal in form and expressed by

$$\begin{pmatrix} A-E & -W & 0 & \cdots & 0 \\ -W & A+W-E & -W & & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & & -W & A+W-E & -W \\ 0 & \cdots & 0 & -W & A-E \end{pmatrix} \quad (6)$$

where $A = A_{11}$; $W = -A_{12}$ and all the other elements of matrix are zeros '0'. In addition, in this tridiagonal form, the matrix ($\underline{A} - E\underline{I}$) can be decomposable in a product of two triangular matrices $\underline{b}$ and $\underline{c}$ given by

$$\underline{b} = \begin{pmatrix} 1 & 0 & \cdots & & 0 \\ b_{12} & 1 & \ddots & & \vdots \\ \vdots & b_{23} & \ddots & 0 & \\ \vdots & & \ddots & 1 & 0 \\ 0 & \cdots & \cdots & b_{N-1,N} & 1 \end{pmatrix} \quad (7)$$

and

$$\underline{c} = \begin{pmatrix} c_{11} & c_{21} & 0 & & \cdots & 0 \\ 0 & c_{22} & c_{32} & \ddots & & \\ \vdots & & & \ddots & c_{N-2,N-1} & \vdots \\ \vdots & & & \ddots & c_{N-1,N-1} & c_{N,N-1} \\ 0 & \cdots & 0 & \cdots & 0 & c_{NN} \end{pmatrix} \quad (8)$$