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The decoherence of the parabolic linear bound potential quantum dot qubit

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Abstract

On the condition of electric–LO phonon strong coupling in parabolic quantum dot, we obtain the eigenenergies of the ground state and the first-excited state, the eigenfuctions of the ground state and the first-excited state by using variational method of Pekar type. This system in quantum dot may be employed as a two-level quantum system—qubit. The phonon spontaneous emission causes the decoherence of the qubit. The relations between the decoherence time with the coupling strength, the confinement length, the coefficient dispersion are discussed.

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Keywords: Quantum dot; Qubit; Decoherence

1. Introduction

Quantum computer has attracted considerable attention in information science field recently. And the quantum computation intended as information theory performed using qubits, that is, two-level systems evolving by the unitary evolution as obtained by the laws of quantum mechanics, demands an increasing control on such systems. Quantum computation is a process of quantization and it is necessary to remain the superposition of quantum state to some extent in the entire computing process. It is required that a qubit be well isolated away from the external environment. However, quantum systems are very frail and the interaction of a quantum memory with its environment destroys the quantum coherence of the stored information, a process called decoherence. Therefore, quantum decoherence plays a very important role in the formalism of quantum computing. So a great deal of considerable efforts [1–4] have been made to investigate the quantum decoherence and how to prolong the decoherence time, in recent years. Several schemes have been proposed for realizing quantum computer. The most feasible approach is the quantum dot because of the advantage of being integrated. The two-level quantum system can be employed as a single qubit in a quantum dot. For this single electron QD qubit, Li et al. [5,6] presented a kind of parameter-phase diagram schemes and defined the parameter region for the use of an InAs/GaAs as a two-level quantum system and using the electronic field prolong the time of decoherence.

We obtain the eigenenergies of the ground state and the first-excited state, the eigenfuctions of the ground state and the first-excited state by using variational method of Pekar type on the condition of electric–LO phonon strong coupling in parabolic quantum dot [7]. This system in quantum dot may be employed as a two-level quantum system qubit. The phonon spontaneous emission causes the decoherence of the qubit. We discuss the relations between the decoherence time with the coupling strength, the confinement length, the coefficient dispersion in this paper.

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2. Theoretical model

The electrons are much more confined in one direction (taken as the Z direction) than in other two directions. Therefore, we shall confine ourselves to taking into account only the effect of electron and LO-phonon and consider only electron moving on the X–Y plane. We assume that the confining potential in a single QD is parabolic.

$$\mathsf{V}_{(\rho)} = \frac{1}{2} m^* \omega_0^2 \rho^2, \tag{1}$$

where m^* is the band mass of electron, ρ is the coordinate vector of a two dimensions and ω_0 is the confinement strength. The Hamiltonian of electron–phonon system is given by

$$\mathsf{H} = -\frac{\hbar^2}{2m^*}\nabla_\rho^2 + \frac{1}{2} m^* \omega_0^2 \rho^2 + \sum_q \hbar\omega_{\mathrm{L0}} b_q^+ b_q, + \sum_q (V_q \mathrm{e}^{\mathrm{i}q.r} b_q + \mathsf{h}\cdot\mathsf{c}). \tag{2}$$

To compute more easily, we choose the units $\hbar = 2m^* = 1$. The Hamiltonian is written as follows:

$$\mathsf{H} = -\nabla_\rho^2 + \frac{1}{4}\omega_0^2\rho^2 + \sum_q \omega_{\mathrm{LO}} b_q^+ b_q + \sum_q (V_q \mathrm{e}^{\mathrm{i}q.r} b_q + \mathsf{h}\cdot\mathsf{c}), \tag{3}$$

where $b_q^+(b_q)$ is the creation (annihilation) operator of bulk LO-phonon with the wave vector $q(q = q_{//}, q_\perp)$, $\mathsf{r} = (\rho, z)$ is the coordinate of the electron and

$$V_q = \frac{\mathrm{i}}{q}(\omega_{LO})^{3/4}\left(\frac{4\pi\alpha}{V}\right)^{1/2}, \tag{4}$$

$$\alpha = \frac{e^2}{2}(\omega_{LO})^{-1/2}\left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right). \tag{5}$$

Using the LLP transformation in Eq. (2)

$$\mathsf{U} = \exp\left[\sum_q \left(f_q b_q^+ - f_q^* b_q\right)\right], \tag{6}$$

where f_q is treated as a variational function, we have

$$H' = U^{-1} H U. \tag{7}$$

Supposing that the Gaussian function approximation is valid in the ground state of electron–phonon system, by variational method of Pekar type we have

$$\left|\varphi_{e-p}\right\rangle = \frac{\lambda}{\sqrt{\pi}} \exp\left(-\frac{\lambda^2\rho^2}{2}\right)\left|\xi(z)\right\rangle\left|0_{\mathrm{ph}}\right\rangle, \tag{8}$$

where λ is the variational parameter, since the electrons are much more strongly confined in Z direction than in other two directions and are considered to be confined in a infinitesimally narrow layer, so $\left\langle\xi(z)\middle|\xi(z)\right\rangle = \delta(z)$. And $\left|0_{\mathrm{ph}}\right\rangle$is the unperturbed zero phonon state which satisfies $b_q\left|0_q\right\rangle = 0$. We then obtain the electron ground-state energy is the following form:

$$E_0(\lambda) = \left\langle\varphi_{e-p}\middle|H'\middle|\varphi_{e-p}\right\rangle = \lambda^2 + \frac{1}{\lambda^2 l_0^4}, - \frac{1}{2}(2\pi\omega_{\mathrm{LO}})^{1/2}\alpha\lambda, \tag{9}$$

where $l_0 = (\hbar/2m^*\omega_0)^{1/2}$. We have the electron ground-state energy to be given by

$$E_0 = \lambda_0^2 + \frac{1}{\lambda_0^2 l_0^4} - \frac{1}{2}(2\pi\omega_{\mathrm{LO}})^{1/2}\alpha\lambda_0. \tag{10}$$

Similarly, the trial wave function of electron–phonon system in the first-excited state may be chosen as

$$\left|\varphi_{e-p}\right\rangle' = \frac{\lambda^2}{\sqrt{\pi}}\rho\,\exp\left(-\frac{\lambda^2\rho^2}{2}\right)\exp(\pm\mathrm{i}\phi) \times \left|\xi(z)\middle|0_{\mathrm{ph}}\right\rangle. \tag{11}$$

This satisfies the following relations:

$$\left\langle\varphi_{e-p}\middle|\varphi_{e-p}\right\rangle' = 0, \quad \left\langle\varphi_{e-p}\middle|\varphi_{e-p}\right\rangle' = 1. \tag{12}$$

We can obtain the energy in the first-excited state by using the $E_1 = \left\langle\varphi_{e-p}\middle|H'\middle|\varphi_{e-p}\right\rangle'$:

$$E_1(\lambda) = 2\lambda^2 + \frac{2}{\lambda^2 l_0^4} - \frac{11}{32}(2\pi\omega_{\mathrm{LO}})^{1/2}\alpha\lambda. \tag{13}$$

The first-excited state energy of electron in a parabolic QD can be written as

$$E_1 = 2\lambda_0^2 + \frac{2}{\lambda_0^2 l_0^4} - \frac{11}{32}(2\pi\omega_{\mathrm{LO}})^{1/2}\alpha\lambda_0. \tag{14}$$

We can obtain λ_0 by using the variational method. Then we can get the eigen level and the eigenwavefunction. Then, we obtain the two-level system needed by a single qubit. The superposition state of electron can be expressed as

$$\left|\psi_{01}\right\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \tag{15}$$

where

$$|0\rangle = \varphi_0(\rho) = \frac{\lambda_0}{\sqrt{\pi}}\exp\left(-\frac{\lambda_0^2\rho^2}{2}\right), \tag{16}$$

$$|1\rangle = \varphi_1(\rho) = \frac{\lambda_0^2}{\sqrt{\pi}}\rho\,\exp\left(-\frac{\lambda_0^2\rho^2}{2}\right)\exp(\pm\mathrm{i}\phi). \tag{17}$$

On the condition of low temperature, we introduce the parabolic phonon dispersion $\omega = \omega_{\mathrm{LO}} - \eta c q^2$. Under the dipole appreciations based on the Fermi Golden Rule, the spontaneous emission rate can be written in the following form:

$$T^{-1} = \frac{e^2\Delta E}{2\pi c\varepsilon_0\hbar^2\eta}\left|\langle 0|\rho|1\rangle\right|^2, \tag{18}$$

where c is the speed of light in vacuum, ε_0 is the material (vacuum) dielectric constant, η is the coefficient dispersion,