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Bound values for Hall conductivity and percolation under quantum Hall effect conditions

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Abstract

The percolation under Quantum Hall Effect conditions in inhomogeneous medium has been studied. The lower and upper bound possible values for effective Hall conductivity values have been established. It has been shown that these bound values for Hall conductivity differ from bound values for metal conductivity. It comes from unusual character of current percolation under Quantum Hall Effect conditions. The physical sense of obtained results has been discussed.

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1. Introduction

In the study of the current percolation in inhomogeneous medium the problem of the effective conductivity of the medium σ_e is appeared. The effective conductivity has been determined as a coefficient of proportionality between average electric current $\vec{J} = \langle \vec{j} \rangle$ and average electric field $\vec{E} = \langle \vec{e} \rangle$

$$\vec{J} = \sigma_e \vec{E}. \quad (1)$$

The effective conductivity has been easily established from boundary conditions for the simple case of layered system, consisting of two alternating layers with different conductivities σ_1 and σ_2 and equal widths $d_1 = d_2$. In the case, when electric current flows perpendicular to the interface of layers, the normal component of electric current is conserved:

$$j_{1n} = j_{2n} = J/2 \quad (2)$$

and the resistance has been averaged:

$$\rho \perp = \langle \rho \rangle = \frac{1}{2}(\rho_1 + \rho_2). \quad (3)$$

In the other case, when electric current flows along layers, the longitude component of electric field is conserved:

$$e_{1t} = e_{2t} = E/2 \quad (4)$$

and the conductivity has been averaged:

$$\sigma^{||} = \frac{1}{2}(\sigma_1 + \sigma_2). \quad (5)$$

In two-dimensional heterogeneous medium some exact results for effective conductivity of the random medium have been obtained due to the dual symmetry. Firstly it was the Keller theorem [1] and secondly a general approach, which has been independently put forward in Dykhne works [2]. It has been shown, that the effective conductivity of two-phase medium at the percolation threshold (at equal phase concentrations) equals to:

$$\sigma_e = \sqrt{\sigma_1 \sigma_2}. \quad (6)$$

The duality relation for effective conductivity has been established at arbitrary phase concentrations:

$$\sigma_e(\varepsilon)\sigma_e(-\varepsilon) = \sigma_1 \sigma_2, \quad (7)$$

where $\varepsilon = X - X_c$ is the deviation from percolation threshold $X_c = 1/2$. (We call the system as dual system

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relative to the initial one if it differs from initial only by the replacement of phases and if it has the same geometric phase placement). This relation has been obtained due to the additional symmetry of the two-dimensional equations for the constant current and Ohm's law relative to rotational transformations (see below Section 3). It has been shown that formula (9) has a sense of stationary point for Dykhne's transformations in Ref. [3]. In the works of [4,5] the local distributions of currents (fields) have been founded for "chess-board" square structures by the method of conformal map additionally.

In the general case the bound values for effective conductivity have been obtained:

$$\left\langle \frac{1}{\sigma} \right\rangle^{-1} \leqslant \sigma_{\mathrm{e}} \leqslant \langle \sigma \rangle. \tag{8}$$

Here $\langle a \rangle$ means the average value of the quantity a. Briefly remind how these results have been obtained [6]. For this aim the expression for Joulle dissipation energy has been used.

$$Q = \frac{1}{V}\int (\vec{j}, \vec{e})\,\mathrm{d}V = \vec{J}\vec{E} = \sigma_{\mathrm{e}}\vec{E}^2. \tag{9}$$

If we insert the value for average current $\langle \vec{j} \rangle = \vec{J}$ in the integral of formula (9), so the lower bound value of effective conductivity in the formula (8) has been obtained . If the value for average electric field $\langle \vec{e} \rangle = \vec{E}$ has been inserted, so the upper bound value of formula (8) has been followed. One can see estimations for bound values of effective conductivity in more details in review [7].

In this paper the percolation under quantum Hall effect (QHE) conditions ($\sigma_{xx} = 0, \sigma_{xy} = \text{const}$) has been studied. Feature of such a percolation under QHE conditions consists of that the Hall current is always directed perpendicular to the electric field:

$$\vec{j} = \sigma_{xy}[\vec{n}, \vec{e}]. \tag{10}$$

Here $\vec{n}$ is an unit vector, which directed along magnetic field and perpendicular to the considered plane. So the dissipation always equals zero under QHE regime: $Q = 0$, that is, the Hall phases are non-dissipative phases. It means also that the we cannot apply above reasonings to obtain the bound values for effective Hall conductivity under QHE conditions and cannot estimate possible values for Hall conductivity of heterogeneous medium by usual way.

Also from standard boundary conditions (2), (4) and the expression for Hall current (10) the new boundary conditions have been obtained:

$$j_{1n} = 0, \quad j_{2n} = 0. \tag{11}$$

The another problem comes from these new boundary conditions, because it seems that there is no a transfer (current) through interface of phases accordingly (11), for exception, only few singular points. Consequently, it seems too that the effective Hall conductivity must be equal to zero everywhere: $\sigma^e_{xy} = 0$.

But as it will be shown below it is not so due to percolation through singular points. In this paper we find the bound values for effective Hall conductivity and explain the physical sense of percolation in composite systems under Quantum Hall Effect conditions. The paper is constructed as follows. In Section 2 the usual layered systems under QHE conditions have been considered. The effective Hall conductivity tensor has been obtained for layered systems. In Section 3 the effective Hall conductivity has been calculated for random two-phase systems. The Dykhne's method of rotational transformations has been used to solve this problem. It was shown that the Hall conductivity bound values have been determined by the connectivity of systems. In Section 4 the simple model of circular metal inclusion has been considered to clarify the physical sense of current percolation under QHE conditions. In Section 5 short discussion of obtained results has been given.

2. Percolation in layered systems under QHE conditions

To understand the features of current percolation under QHE conditions let us consider the simple model of layered media, consisting of two alternating layers with equal widths and different Hall conductivities $\sigma^{(1)}_{xy}$ and $\sigma^{(2)}_{xy}$. Let us assume all layers are directed along y-direction.

In the case when electric current flows perpendicular to the phase interfaces, the electric field directs along layers according (10) for Hall current and it equals to the average value. It has been followed from boundary conditions (4). Correspondingly, we obtain the following formulae for effective Hall conductivity in this case:

$$\sigma^e_{xy} = \langle \sigma_{xy} \rangle = \frac{1}{2}(\sigma^{(1)}_{xy} + \sigma^{(2)}_{xy}). \tag{12}$$

To check this result we calculate the distributions of electric fields and currents at every phase, using the definitions for averaged electric field and averaged electric current:

$$\sigma^{(1)}_{xy}\langle [\vec{n}, \vec{e}] \rangle_1 + \sigma^{(2)}_{xy}\langle [\vec{n}, \vec{e}] \rangle_2 = \sigma^{(e)}_{xy}[\vec{n}, \vec{E}],$$
$$\langle \vec{e} \rangle_1 + \langle \vec{e} \rangle_2 = \vec{E}. \tag{13}$$

After simple calculations we obtain the formula for electric fields (currents) at every phase:

$$\langle \vec{e} \rangle_1 = \vec{E}\,\frac{\sigma^{(e)}_{xy} - \sigma^{(2)}_{xy}}{\sigma^{(1)}_{xy} - \sigma^{(2)}_{xy}}, \quad \langle \vec{e} \rangle_2 = \vec{E}\,\frac{\sigma^{(1)}_{xy} - \sigma^{(e)}_{xy}}{\sigma^{(1)}_{xy} - \sigma^{(2)}_{xy}}. \tag{14}$$

Inserting the formula (12) into expressions (14) it is easy to see that

$$\langle \vec{e} \rangle_{1y} = \langle \vec{e} \rangle_{2y} = E_y/2. \tag{15}$$

So we find the solution with constant electric field.

It is necessary to clear the physical sense of obtained result for effective conductivity (12). Because according the boundary conditions in the case of QHE conditions $j_{1n} = 0$, $j_{2n} = 0$ the Hall edge current cannot cross