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Influence of gain localization in one-dimensional random media

Yulong Tang *, Yong Yang, Jianqiu Xu

Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Science, Shanghai 201800, China

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ABSTRACT

Based on the transfer matrix method, we studied the effects of gain distribution on the performance of light localization in one-dimensional completely random systems. Due to the scattering of the pump light, the gain can also be localized. Compared with homogenous gain distribution, the localized gain can induce light confinement more efficiently. The overlapping between the laser modes and the localized gain brings on the high-*Q* modes that are amplified preferentially, and reduces the lasing threshold. The threshold can be hundreds times lowered in some special cases. With increase of the random system size, the localization position goes deeper into the media first, and then saturates when the system is thicker than 550 layers. For a given kind of random laser systems, an optimal system size exists, for which the mode intensity is significantly enhanced. The influence of the system structure on the lasing intensity and localization position is also discussed.

1. Introduction

The amplification of stimulated emission in random scattering media was first proposed by Letokhov in 1968 [1], but the experimental observation of random laser emission was carried out till 1986 from the polycrystalline powder [2], and 1994 from the dye solution [3], respectively. Recently, coherent random laser emission was also extensively observed [4–6]. Since being reported, random lasers have attracted more and more concerns for their unique properties and potential applications. Differing from the conventional lasers, the feedback in the random laser comes from multiple scattering in disordered media rather than from the traditional cavity that consists of mirrors. There are two categories of random lasers: incoherent and coherent. The incoherent random laser, characterized by optical power feedback, occurs in the weakly scattering regime. On the contrary, the coherent random emission, characterized by amplitude feedback, was observed only in the strongly scattering media, where the scattering of light can form closed loop paths [7]. Anderson localization of light was originally considered as the underlying mechanism of coherent random laser in strongly scattering media [8]. Recently, several models such as amplification of quasi-modes [9], random resonators [10], and enhanced amplified spontaneous emission (ASE) [11] have been suggested to explain the coherent random emission.

Many theoretical analyses and numerical simulations have been constructed to understand the unusual behavior of emission observed in random systems [4,5,12]. The analysis methods include the diffusion equation with gain [13], the ring laser with non-resonant feedback [14], the analytical model based on quasi-states [8], and the finite-difference time-domain (FDTD) method combined with the interplay between localization and amplification [15]. Compared with these methods, the transfer matrix (TM) method contains no simplified approximation, and can simulate exactly the wave propagation in random systems [16–18]. Besides, the TM method can calculate the quasi modes of weakly scattering systems that overlap spectrally and have short lifetimes.

Due to the resonance of localized modes, the transmission spectra of localized 1-D systems exhibit many randomly distributed high-transmission peaks [19], which is related to both the system size *L* [20] and the particle density [21]. Besides localized modes, non-localized modes (so-called necklace states) can also exist in 1-D random systems [22]. For generating the confinement of light, the excitation methods can be optical or electrical [23].

In the research of random lasers, most of attention is concentrated on the localization of the lasing light, few researches considered the influence of the localization of pump light. In their simulations, the pump light and gain were assumed to be uniform and homogeneous inside the whole random media. Actually, because the pump light is scattered too by the random media,

* Corresponding author. Tel.: +86 21 69918602; fax: +86 21 69918507.
E-mail addresses: brucetyl@126.com (Y. Tang), jqxu@siom.ac.cn (J. Xu).

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both the pump and lasing light can be trapped inside the strongly scattering media.

In this paper, we explore the interaction between the localized gain and the random emission in one dimensional random media. Based on the TM method, the localizations of both the pump and signal light are calculated. Compared to the homogenous gain, the coupling between the localized pump and signal light enhances the amplification of the localized modes, and reduces the lasing threshold. Lasing features of the random structure with localized gain are examined, and the influence of random system thickness is analyzed.

2. Theoretical model

As shown in Fig. 1, the 1-D random system is made up of a number of dielectric layers. Each layer is constituted by a couple of film a and b. The thickness of the film a and b are randomized by $L_a = L_{a0}(1 + A_a r_a)$ and $L_b = L_{b0}(1 + A_b r_b)$, where r_a and r_b are random numbers distributed between $(-0.5, 0.5)$ uniformly, and the amplitudes of randomness are $0 \leqslant A_a \leqslant 1$ and $0 \leqslant A_b \leqslant 1$. The dielectric constants of the films are $\varepsilon_a = \varepsilon' - \mathrm{i}\varepsilon''(\omega)$ and $\varepsilon_b = 1$, respectively. The whole system is embedded in a homogeneous infinite material with the dielectric constant $\varepsilon_0 = 1$.

For the 1-D case, the time-independent Maxwell's equations are given as (only considering one direction of the electromagnetic field)

$$\begin{cases} \dfrac{\partial^2 E(z)}{\partial z^2} + \dfrac{\omega^2}{c^2}\varepsilon(z)E(z) = 0, \\ \dfrac{\partial^2 H(z)}{\partial z^2} + \dfrac{\omega^2}{c^2}\varepsilon(z)H(z) = 0. \end{cases} \tag{1}$$

The corresponding solution can be written in the form

$$\begin{cases} E(z) = U(z)\exp(\mathrm{i}kz), \\ H(z) = V(z)\exp(\mathrm{i}kz), \end{cases} \tag{2}$$

where $U(z)$ and $V(z)$ are slowly varying amplitudes of electric and magnetic fields, respectively.

Substituting Eq. (2) into Eq. (1) and using the continuity boundary conditions of the electric and magnetic fields, we obtain [24]

$$\begin{bmatrix} U_{n+1} \\ V_{n+1} \end{bmatrix} = M_n \begin{bmatrix} U_n \\ V_n \end{bmatrix}. \tag{3}$$

The characteristic TM of the nth dielectric film layer is

$$M_n = \begin{bmatrix} \cos(kL_n \cos\theta) & -\frac{\mathrm{i}}{p}\sin(kL_n \cos\theta) \\ -\mathrm{i}p\sin(kL_n \cos\theta) & \cos(kL_n \cos\theta) \end{bmatrix}, \tag{4}$$

where $k = (\omega/c)\sqrt{\varepsilon}$, $p = \sqrt{\varepsilon/\mu}\cos\theta$, and θ denoting the angle between z axis and the direction of the co-phasal surface. In active media, the complex dielectric constant (ε) has a passive imaginary part ($\varepsilon'' > 0$), originating from population inversion. When $\varepsilon'' > 0$, the emission light is amplified by the stimulated polarization; while $\varepsilon'' < 0$, the emission light is absorbed. For the whole system, the TM will be $M(L) = \prod M_n$. Using the TM, final electric and magnetic fields intensities can be obtained for given initial fields after propagating in active media.

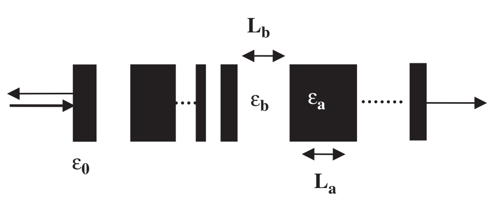

Fig. 1. Diagram of one-dimensional random system.

When pump light travels in the random system, it undergoes scattering and absorption. The medium ions are thus excited by absorbed pump light. The pump rate P_r can be written as $P_r = \eta_a P_{in}/h\nu_p$, where P_{in} is the incident pump power, ν_p is the frequency of pump beam, h is the Plank's constant, and η_a is the fraction of incident pump power absorbed by the random medium. Considering a four-level system, the second level (N_2) and the first level (N_1) are called the upper and the lower lasing levels. The lifetimes of up levels N_3, N_2 and N_1 are τ_{32}, τ_{21} and τ_{10}. The rate equations can be written as [25]

$$\begin{cases} \dfrac{\mathrm{d}N_3}{\mathrm{d}t} = P_r N_0 - \dfrac{N_3}{\tau_{32}}, \\ \dfrac{\mathrm{d}N_2}{\mathrm{d}t} = \dfrac{N_3}{\tau_{32}} - \Delta N \cdot \sigma\phi c - \dfrac{N_2}{\tau_{21}}, \\ \dfrac{\mathrm{d}N_1}{\mathrm{d}t} = \dfrac{N_2}{\tau_{21}} + \Delta N \cdot \sigma\phi c - \dfrac{N_1}{\tau_{10}}, \\ \dfrac{\mathrm{d}N_0}{\mathrm{d}t} = \dfrac{N_1}{\tau_{10}} - P_r N_0, \end{cases} \tag{5}$$

where σ is the stimulated cross section, ϕ is the photon density, c is the velocity of light, and $\Delta N = N_2 - N_1$ is the population inversion per unit volume. In Eq. (5), the contribution of the spontaneous emission is included in the parameter τ_{21}. By solving the rate equations, the population inversion ΔN can be obtained.

In the media, the polarization driven by absorbed pump light $E(\omega)\exp(\mathrm{i}\omega t)$ can be described by the differential equation of an electric oscillator [26] with amplitude of x, frequency of ω_0, and damping constant of γ

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 2\gamma\frac{\mathrm{d}x}{\mathrm{d}t} + \omega_0^2 x = -\frac{e}{m}E(\omega)\mathrm{e}^{\mathrm{i}\omega t}, \tag{6}$$

where e and m are the electron charge and mass, respectively. From Eq. (6), the imaginary part of the complex dielectric constant can be solved as

$$\varepsilon''(\omega) = \frac{\Delta N e^2}{2\omega_0 m} \cdot \frac{\gamma}{(\omega - \omega_0)^2 + \gamma^2}. \tag{7}$$

In terms of light wavelength, $\varepsilon''(\omega)$ is re-written as

$$\varepsilon''(\lambda) = C_0 \cdot \frac{\omega_g \lambda_g^3}{(\lambda - \lambda_g)^2 + \omega_g^2}, \tag{8}$$

where $C_0 = \Delta N e^2/8\pi^2 mc^2$, λ is light wavelength, λ_g is the light wavelength at the center of the gain line, and ω_g is the half-width of the gain spectrum.

3. Numerical simulation and results

Our simulation process is as follow. First, the pump light distribution in the medium is calculated with the TM. Different pump light intensities cause different pump rates in different film layers. Secondly, population inversion ΔN of different film layers is obtained by solving the rate equations of (5), leading to different ΔN dependent ε'' of the complex dielectric constant. Finally, ε'' is substituted into the TM to calculate signal light intensity.

Parameters taken in our simulations are: $R_e(\sqrt{\varepsilon_a}) = 1.82$ and $\varepsilon_b = 1$ (corresponding to Nd:YAG and air, respectively); $L_{a0} = 100\,\mathrm{nm}$, $L_{b0} = 50\,\mathrm{nm}$; $A_a = 0.5$, $A_b = 0.7$; and $\lambda_g = 1064\,\mathrm{nm}$, $w_g = 0.45\,\mathrm{nm}$.

In the 1-D model shown in Fig. 1, light may experience significant reflection from multiple dielectric layers due to the wave interference effect. Collective multiple-scattering interference effects lead to large field enhancement in the random