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Metamaterials and superresolution: From homogenization to rigorous approach

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Abstract

The paper touches upon the correspondence between the widely used homogenized approach and integral equation technique in simulating electromagnetic excitation of metamaterials. The attention is drawn to metamaterials with helix-type inclusions, their properties are briefly discussed. Special attention is paid to the thin metamaterial sheets as they are the primary candidates to manufacture the so-called "superlenses" that can reveal the resolution overcoming the well-known diffraction limit (about half a wavelength). Experimental results are compared to computer simulation using both homogenized model and a rigorous approach. A physical interpretation is suggested of the development of an image with superresolution in a real device.

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1. Introduction

Modern metamaterials with negative values of permittivity and permeability may be manufactured by using inclusions of different shapes. Here we would like to draw the reader's attention to easy to manufacture helix-type inclusions which were investigated earlier [1] and to the experimental results [2], which have demonstrated the effect of superresolution predicted previously by Pendry [3] in Veselago's lens [4], Fig. 1a. The experimental "superlens" was provided by a plate of a composite material filled with resonant elements—spirals with a small pitch and linear half-wave segments of copper wire (see Fig. 1b), excited by the magnetic and electric components, respectively, of a field generated by two linear wire radiators [2].

In Ref. [2] as well as in other works [5–7], the reasons were outlined for the restriction of the limiting attainable resolution of the system originating from introducing negative permittivity and permeability, i.e. from the homogenization principle. However, despite the perfect correspondence of the experimental results and theoretical considerations [2] and general progress in the investigation and understanding of the effects demonstrating unusual electromagnetic response of metamaterials, a lot of questions remain unanswered (Fig. 1). Particularly, the details of the field interaction with thin metamaterial plates consisting of very few rows of resonators are not yet clear, since widely used homogenized models cannot yield reliable results in this case. For example, it was shown in Ref. [8] that, in the case of composites containing extended resonant inclusions, the effective permittivity may be introduced only for sheet materials whose thickness exceeds some critical value and, generally speaking, the value of permittivity may differ depending on the experimental conditions (see also the discussion in Ref. [9]). The experimental determination of the effective parameters of composites containing resonant inclusions is based, as a rule, on the results of measurements under incidence of a wave with a quasi-plane front [10].

2. Numerical modeling

We are going to demonstrate some characteristic features of electromagnetic wave propagation in a real

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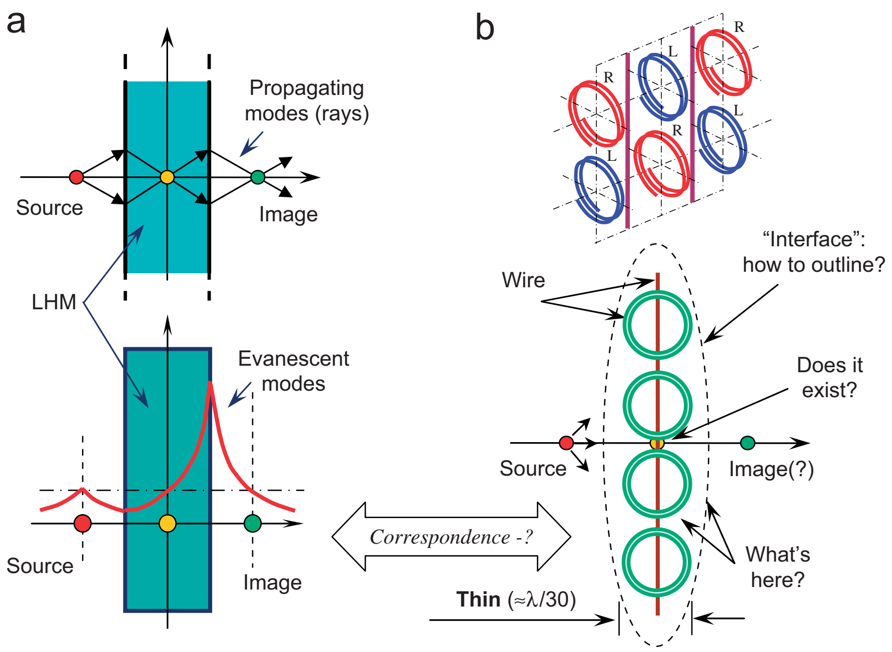

Fig. 1. Homogenized left-handed medium plate and realistic metamaterial.

composite and expose the aspects that turn out to be hidden when the effective parameters are used in Maxwell's equations. This appeared to be possible by applying a rigorous approach (integral equation method) to obtain a full-wave solution for the electromagnetic fields. The system of integral equations of Pocklington's type [11] was solved numerically for a detailed study of interaction between the electromagnetic wave and the composite material. Such a solution of the problem in a rigorous formulation enables one to reproduce all details of the electromagnetic processes occurring in the experimental setup, while completely taking into account the interaction between all wire resonators that make up the experimental plate. The equations are based on a thin-wire approximation with regard for the finite conductivity of the wire metal,

$$ZI + \mathrm{i}\omega\mu_0 \int_L I\left(\vec{v}\cdot\vec{v}' - \frac{1}{k_0^2}\frac{\partial^2}{\partial v\,\partial v'}\right)\frac{\mathrm{e}^{-\mathrm{i}kr}}{4\pi r}\,\mathrm{d}l = \vec{v}\cdot\vec{E}^{\mathrm{i}},$$

where $L = \sum L_i$, $Z = (1+i)/2\pi a\sigma d$.

Such thin-wire equations were extensively studied, and details regarding the kind of approximation, corresponding limitations, solution techniques, etc. can be found in numerous textbooks, see, for example, Refs. [12–14]. Each equation is set up relative to the linear current density I in wire elements L_i of a composite; here, $\vec{v}$ and $\vec{v}'$ are unit vectors of tangents to the wire axis at observation and integration points, respectively, r is the distance between these points, and $\vec{E}^{i}$ is the vector of intensity of incident field of frequency ω. The finite conductivity of the wire metal including the skin effect was taken into account by introducing its specific (per unit length) impedance Z (d is the skin depth, and a and σ denote the wire radius and conductivity of its material, respectively). The numerical algorithm to solve this integral equation was developed using the widely employed Galerkin method (i.e., the moment method with roof-top expansion and weighting functions) and was thoroughly tested [15]. The scattered field was calculated by the obtained currents using vector potentials.

Discussed below are the results of numerical simulation obtained for a composite plate with a finite number of elements that corresponds to that of a real experimental sample [2]. The calculation results both reproduced the observed effect of superresolution in the presence of a complex composite medium, and made it possible to compare the phenomena occurring in real samples of composites (periodic systems of resonant elements) to phenomena occurring in homogeneous media with negative electrodynamic parameters (metasubstances) that exist only theoretically.

It was demonstrated that a plate of a composite exhibits some properties typical of a plate of metasubstance. For example, a frequency band exists (as predicted by theory, it is located in the vicinity of and a little higher than the resonance frequency of inclusions) in which the effect of superresolution shows up (see Fig. 2a). For comparison, Fig. 2b gives analogous pattern in the absence of a plate, that is, an incident field pattern (all geometrical dimensions along the graph axes are given in electrical units, i.e., are multiplied by $k = 2\pi/\lambda$, where λ is the wavelength in free space).

By and large, the composite plate may be characterized as a device in which a backward wave exists, i.e., there is a zone of space in the vicinity of resonators in which the