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Physica B 394 (2007) 256–261
PHYSICA B
www.elsevier.com/locate/physb

Dynamic mass density and acoustic metamaterials

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Abstract

Mass density of a composite is generally taken as the volume-averaged value of components' densities. Moreover, the same volume-averaged mass density is usually used to calculate the wave speed in the long-wavelength limit, i.e., where the wavelength is much larger than the size of the inhomogeneities. In this paper, we show via rigorous derivation that the dynamic mass density used in the calculation of (long-wavelength) wave speed can differ significantly from the static volume-averaged value. This recognition is shown to yield an excellent account of some recent experimental data, as well as to make possible the realization of acoustic metamaterials. Physical reason for the difference between two mass densities is attributed to the relative motion between the components. That is, the implicit assumption—that all components in a composite must move uniformly in the long-wavelength limit—can be violated in the limit of large acoustic impedance contrast between the components. The dynamic mass density can even be negative for the locally resonant sonic materials as demonstrated both experimentally and theoretically. The implications of this finding, in the context of acoustic metamaterials, are discussed.

PACS: 43.35.+d; 43.20.+g; 43.40.+s

Keywords: Acoustic metamaterials; Mass density; Local resonance; Effective medium; Composites; Wave propagation

1. Introduction

It is regarded as common sense that the mass density of a composite must be the volume-averaged densities of its components, e.g., $D_V = (1-f)D_1 + fD_2$ for a two-component composite in which component 1 constitutes the matrix and component 2 the dispersed inclusions, with respective densities D_1, D_2, and f being the filling ratio of the inclusions. This expression is denoted below as the volume-averaged mass density (VAMD, or D_V). Indeed, the static version of the mass density must be the VAMD, since it is verifiable through simply weighing the composite and its constituents, and measuring the respective volumes. However, an important application of the composite mass density is in the prediction of wave speed v at the low-frequency limit, where the relevant wavelength is much larger than the typical feature sizes in the composite, i.e., $v = \sqrt{M/\rho}$, where M is the effective modulus of the composite and ρ is defined to be the dynamic mass density. The question is: does $\rho = D_V$ necessarily? As explained below, the answer to this question has a direct bearing on the realizability of acoustic metamaterials.

Historically, it has always been assumed that $\rho = D_V$, except two decades ago Berryman [1] derived a different dynamic mass density expression for the prediction of (fluid matrix—solid) composite wave properties in the long-wavelength limit, based on the average T-matrix approach:

$$\frac{\rho - D_1}{(d-1)\rho + D_1} = f\frac{D_2 - D_1}{(d-1)D_2 + D_1}, \tag{1}$$

where d denotes the spatial dimensionality of the problem. The Berryman effective mass density expression is noted to differ significantly from the intuitive VAMD, and for all the intervening years after the initial derivation it has remained a curiosity rather than extensively used, mainly owing to the lack of experimental support as well as to the strong sense that the intuitive VAMD must be correct, since otherwise it would be equivalent to stating the rather

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doi:10.1016/j.physb.2006.12.046

radical principle that the static mass density for a composite should be different from its dynamic mass density, even in the long-wavelength limit. An additional objection to the Berryman expression is that the derivation treats the multiple scatterings inherent in the inhomogeneous system only in an averaged sense, and therefore not rigorous.

It is the purpose of this paper to clarify the relationship between the dynamic and static mass densities, and to delineate some of the implications of our findings [2]. In what follows, a description of the rigorous derivation approach is followed by the application of the result to some recent experimental data, together with a physical explanation of the difference between ρ and D_V in that particular case. The possibility of a negative ρ is then explored and demonstrated in the case of locally resonant sonic materials [3]. We conclude with a discussion on the possible experimental realization of acoustic metamaterials.

2. Rigorous derivation approach

Since our goal is to derive an expression for the dynamic mass density of a composite, the starting point is necessarily the elastic wave equation. As a composite is usually characterized by random microstructures, the wave equation cannot be solved exactly. Hence, the conventional approach to the derivation of so-called "effective media" equations necessarily involves approximations, such as the "average T-matrix approximation" or the "coherent potential approximation". Such approximations are usually in the nature of treating the multiple scatterings in some "average" manner.

Here, we propose a different, rigorous approach to the derivation of the effective media equations in the long-wavelength limit. In order to be specific, let us take the example of a two-dimensional (2D) problem involving aligned cylinders dispersed in a matrix. The starting point of our approach is to treat a periodic microstructure, i.e., the cylinders are placed on a 2D lattice. Due to the periodicity, exact solution of the wave equation becomes possible. Multiple scattering theory [4–6] (MST) is one such exact approach, which will be described below. Given MST, we can always take the long-wavelength limit of the relevant equations by letting the wave frequency $\omega \to 0$. In that limit the dispersion relation of the wave (in the periodic microstructure) must be linear, because one wavelength covers many periods of the structure, hence losing its wave resolution, and the periodic composite would appear homogeneous to the probing wave. The slope of the linear dispersion relation yields the desired effective wave speed.

But how can such an approach be exact for a composite in which the aligned cylinders are randomly dispersed? Does not the answer depend on which periodic microstructure is chosen? To answer such questions, it is only necessary to observe that the structure of the system enters only in the higher order expansion of ka, where k is the wave vector and a the lattice constant. In another words, to the leading order the slope of the linear part of the dispersion relation is independent of the periodic structure, but depends only on the filling factor f. Therefore, whether the structure is random or periodic does not matter. The effective wave speed obtained by taking the long-wavelength limit of the MST thus represents that of the random microstructure as well! In this manner, we are able to capture rigorously all the MSE in our effective medium expression.

Of course, there are limits to such an approach. What is described above can work only to the extent that the randomness does not introduce new statistical correlations, such as connectivity of the system, that are absent in the original problem.

3. Multiple scattering theory

MST accounts fully for all the multiple scattering effects between any two scatterers, plus the vector character of elastic waves, without any approximation. For our particular case of 2D elastic MST in polar coordinates, the displacement $\vec{u}$ of the incident wave on scatterer i and the scattered wave (by the same scatterer) may be expressed, respectively, as

$$\vec{u}_i^{\,\mathrm{in}}(\vec{\rho}_i) = \sum_n a_n^i \vec{J}_n^{\,i}(\vec{\rho}_i),$$

$$\vec{u}_i^{\,\mathrm{sc}}(\vec{\rho}_i) = \sum_n b_n^i \vec{H}_n^{\,i}(\vec{\rho}_i), \tag{2}$$

where the vector functions $\vec{J}_n(\vec{\rho})$ and $\vec{H}_n(\vec{\rho})$ are defined as

$$X\vec{J}_n(\vec{\rho}) = \nabla[J_n(\alpha_1\rho)\mathrm{e}^{\mathrm{i}n\varphi}],$$

$$\vec{H}_n(\vec{\rho}) = \nabla[H_n(\alpha_1\rho)\mathrm{e}^{\mathrm{i}n\varphi}], \tag{3}$$

with α_1 being the wave number in the fluid matrix, $\vec{\rho} = (\rho, \varphi)$ denotes the polar coordinates, and $J_n(x)$ and $H_n(x)$ on the right side of Eq. (3) denote the nth-order Bessel function and Hankel function of the first kind, respectively. Since the incident wave on scatterer i comes from the scattered waves by all the scatterers except scatterer i, we have

$$\vec{u}_i^{\,in}(\vec{\rho}_i) = \sum_{j \neq i} \sum_{n''} b_{n''}^j \vec{H}_{n''}^{\,j}(\vec{\rho}_j). \tag{4}$$

With the help of the addition theorem, we can prove that

$$\vec{H}_{n''}^{\,j}(\vec{\rho}_j) = \sum_n G_{n''n}^{ij} \vec{J}_n^{\,i}(\vec{\rho}_i), \tag{5}$$

where $G_{nn'}^{ij} = G_{nn'}(\vec{R}_j - \vec{R}_i)$ denotes the translation (from scatterer i to scatterer j) coefficients, with $\vec{R}_{i(j)}$ denoting the position of scatterer $i(j)$. We refer to Ref. [6] for the precise definition of $G_{nn'}(R)$. For a given scatterer, the scattered field is completely determined from the incident field