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Electrically controlled dispersion in a nematic cell

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Abstract

In this work, we show theoretically how the trajectories of a propagating optical beam traveling in a planar-homeotropic hybrid nematic crystal cell depend on the wavelength of the optical beam. We apply a uniform electric field perpendicular to the cell to modify these trajectories. The influence of both, the electric field intensity and the refraction index dependence on the wavelength, give rise to an electrically tuned dispersion that may be useful for practical applications.

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1. Introduction

Conventional instruments designed to separate light into their chromatic components are based on the dispersion phenomena that can be produced either, by a set of prisms or by a periodical media such as diffraction gratings that amplify the difference between the optical paths of the chromatic components coming from a polychromatic beam. For the first case, the wavelength selection depends on the geometry of the arrangement given by a combination of prisms and apertures that have to be rotated to select the desired wavelength. Similar procedures have to be used for the diffraction grating where the spatial resolution of the device depends on the shape and periodicity of the diffraction grating and also on the materials used to fabricate them [1].

In recent years, liquid-crystal technology has been applied to fabricate diffraction gratings that can modulate the diffraction efficiency through the use of electric fields [2] which distort the orientational configuration of the liquid crystal by producing a modulation of the refractive index. However, this technology has the disadvantage of giving a different direction for each outgoing chromatic component. This fact compels the user to rotate the system for having a normally incident beam on a CCD detector for maximizing the signal.

Liquid crystals are anisotropic materials that can be used to produce a continuous gradient of refractive index by a proper treatment of their confining surfaces. These surface-induced gradients can be controlled by the application of electric or magnetic fields. It has been shown [3] that for a monochromatic beam impinging obliquely onto a nematic hybrid cell, the optical range can reach lengths several times larger than the cell's thickness. Hence, it is possible to use it as a dispersive media similar to the glass prism. In contrast to the devices discussed above, this nematic slab has the advantage that all the outgoing beams corresponding to different wavelengths will emerge parallel to each other. Furthermore, since the orientation can be controlled by electric fields, it is possible to electrically manipulate the thickness of a polychromatic beam to tune the resolution of the device.

In this work, we calculate the optical path for a polychromatic beam and show how the dispersion phenomenon occurring inside the nematic cell could be used to

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develop a practical scheme to split light into their chromatic components based on the electrically controlled dispersion we present here.

2. Calculation

The system under study consists of a pure thermotropic nematic confined between two parallel substrates with refraction indices N_t and N_b, respectively, as shown in Fig. 1. The cell thickness, l, measured along the z-axis, is small compared to the dimension, L, of the cell plates. The director's initial configuration is spatially homogeneous along the plane x–y and varies with z as given by

$$\hat{\mathbf{n}} = [\sin\theta(z), 0, \cos\theta(z)], \tag{1}$$

which satisfies the hybrid boundary conditions

$$\theta(z=0) = 0,$$
$$\theta(z=l) = \frac{\pi}{2}, \tag{2}$$

where $\theta(z)$ is the orientational angle defined with respect to the z-axis.

A low-frequency uniform electric field E_0, parallel to the z-axis is applied. Then, the equilibrium orientational configurations of the director's field are specified by minimizing the total Helmholtz free energy functional as

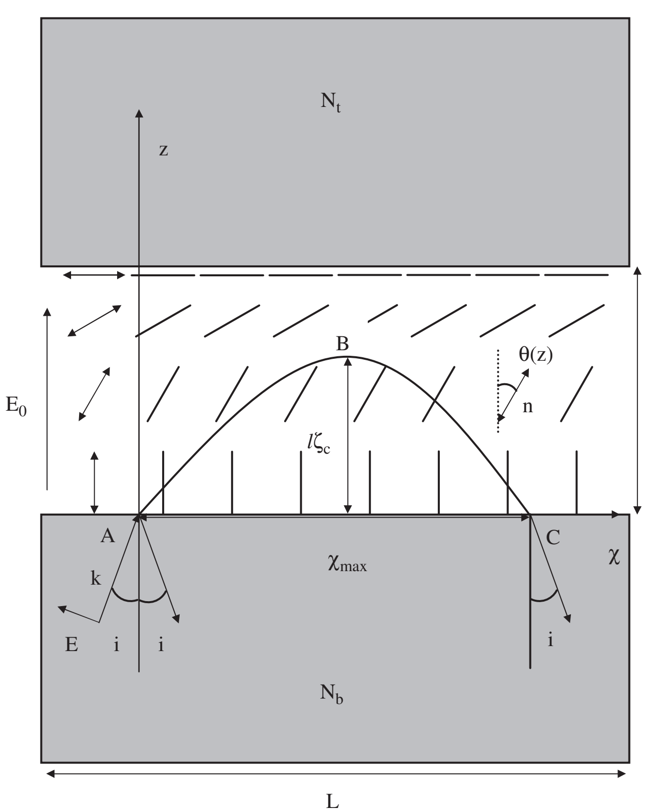

Fig. 1. Illustration of a pure thermotropic nematic confined between two parallel substrates. A P-polarized mode is traveling along the hybrid nematic. $N_b, N_t > n_\parallel, n_\perp$. The trajectory of the beam shows a caustic. ζ_c is the ray penetration. We have introduced the dimensionless variables $\zeta \equiv z/l$ and $\chi \equiv x/l$.

shown in Ref. [4]. We consider a uniaxial medium for which the dielectric tensor ε_{ij} has the general form

$$\varepsilon_{ij} = \varepsilon_\perp \delta_{ij} + \varepsilon_a n_i[\theta(z)] n_j[\theta(z)], \tag{3}$$

where $\varepsilon_\perp$ and $\varepsilon_\parallel$ are the dielectric constants perpendicular and parallel to the director and $\varepsilon_a \equiv \varepsilon_\parallel - \varepsilon_\perp$ is the dielectric anisotropy. Also, we shall assume the equal elastic constants approximation in which the elastic constants associated with the splay, twist, and bend deformations are described by a single constant K. Then, the free energy functional turns out to be [4]

$$F = \int_V \mathrm{d}V \left[\frac{1}{2} K \left(\frac{\mathrm{d}\theta}{\mathrm{d}z} \right)^2 - \frac{E_0^2}{8\pi} (\varepsilon_\perp + \varepsilon_a \cos^2\theta(z)) \right]. \tag{4}$$

The first and second terms of this equation represent the elastic and electromagnetic contribution of the free energy densities, respectively. The stationary configuration is then obtained from the corresponding Euler–Lagrange equation which reads

$$\frac{\mathrm{d}^2\theta(\zeta)}{\mathrm{d}\zeta^2} - q \sin 2\theta(\zeta) = 0. \tag{5}$$

Here, we have used the dimensionless variable $\zeta \equiv z/l$ and the parameter $q \equiv \tilde{\varepsilon}_a V^2 / 8\pi K$ which denotes the ratio between the electric energy and the elastic energy densities; in this sense, it measures the coupling between the electric field and the nematic. Here $\tilde{\varepsilon}_a$ is the low-frequency dielectric anisotropy and $V \equiv E_0 l$ is the applied voltage.

An obliquely incident light beam whose polarization is contained in the incidence plane $x - z$ (linearly P-polarized), impinges the nematic with an angle of incidence i as shown in Fig. 1. We shall assume that the intensity of the beam is low enough such that it does not distort the nematic's configuration. Thus, the dynamics of this optical field is described by the corresponding Maxwell's equations containing the dielectric tensor ε_{ij}, Eq. (3), which depends on θ. The procedure to solve these equations has been carried out in detail for a hybrid cell similar to the one considered here [5] and it was found that there is a regime (so-called second regime) for the angle of incidence i where the ray trajectory exhibits a caustic, that is, where it bends and remains inside the cell until it returns back towards the incidence substrate (see Fig. 1). This trajectory is given by [5]

$$\upsilon = \chi - \int_0^\zeta \mathrm{d}\eta \frac{\varepsilon_{xz} \mp p\sqrt{\varepsilon_\perp \varepsilon_\parallel}/\sqrt{\varepsilon_{zz} - p^2}}{\varepsilon_{zz}}. \tag{6}$$

In this equation the dimensionless variable $\chi \equiv x/l$ has been introduced and $p \equiv N_b \sin i$ is the ray component in the x direction. Here υ is a constant to be determined by the coordinates of the point of incidence of the beam, that is, it establishes an initial condition. The $\pm$ sign in Eq. (6) corresponds to a ray traveling with $\mathbf{k}$ in the $\pm z$ direction, that is, going from A to B and from B to C, respectively, where the point B is the turning point and whose penetration length is ζ_c (see Fig. 1).