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Simple theoretical analysis of the interband optical absorption coefficient in wide-gap semiconductors in the presence of an external electric field and its dependence on a longitudinal magnetic field

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Abstract

An attempt is made to present a simplified theoretical analysis of the interband optical absorption coefficient (OAC) due to a constant uniform electric field on the basis of the electron wave vector dependence of the optical matrix element (OME) for the incident photon energy, ($\hbar\omega$), below and above the band-gap (E_g). It has been found, taking n-GaAs as an example for numerical computation, that the expression of the OAC exhibits an exponential fall-off with the electric field and the photon energy without the consideration of the Wannier–Stark levels, which generally exists in a band due to the external electric field. The effect of a longitudinal magnetic field on the OAC is also studied on the basis of the fact that the transverse wave vector ($k_\perp$) is quantized due to parallel magnetic field and is conserved in the interband optical transition of electrons. Similar results, such as singularity for the case $\hbar\omega < E_g$ in the OAC and the oscillations in the OAC for the case $\hbar\omega \geqslant E_g$in presence of electric field are also obtained with modifications in the expressions for the OAC in presence of electric and parallel magnetic fields. Present study explains the modification of the band-gap of the semiconductor in the presence of electric and parallel magnetic fields, respectively. The numerical analyses are performed and discussed in details taking n-GaAs as an example.

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1. Introduction

In recent years, there has been considerable interest in studying the influence of magnetic field on the interband optical absorption in solids due to a uniform electric field [1,2]. The present work describes a more quantitative approach of magneto-optics for semiconductors whose energy band structure in the presence of magnetic field can adequately be described within the framework of Yafet [3].

The effect of an electric field on optical absorption in semiconductors, near the absorption edge, was investigated in early works by Franz [4] and Keldysh [5]. These authors have shown that in the presence of an electric field, the absorption occurs for photon energies lower than E_g. This phenomenon is known as Franz–Keldysh effect and has been experimentally confirmed [6,7]. It may be noted that the effect of an electric field on the band structure of a solid is needed for the interpretation of the experimental electro-optic observation. This has gained momentum after Wannier [8], who predicted that a ladder like splitting of the band levels of Bloch electrons occurs due to the presence of a uniform external electric field. This ladder-like structure of the band is known as "Wannier–Stark ladders (WSL)". The existence of WSL has been found in the experimental work of Koss and Lambert [9]. It may be

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noted that WSL also exists in semiconductor band structure in the presence of an electric and a longitudinal magnetic field [10]. In addition to the lowering of the optical band-gap of the semiconductor (i.e., the Franz–Keldysh effect) in the presence of an electric field, one expects oscillations in the OAC in the presence of an external electric field, due to WSL. Callaway [11] for the first time predicted such oscillations in the OAC in the presence of an external electric field, considering WSL. The oscillations occur for the incident photon energy ($\hbar\omega$) both below and above E_g. The Franz–Keldysh effects are also observed in semiconductors both theoretically and experimentally [12,13] in the presence of electric field and parallel magnetic field using the effective mass approximation (EMA) techniques by using the non-realistic assumption that the optical matrix element (OME) is independent of the electron wave vector ($\bar{k}$).

Callaway formulated an expression for the interband transition probability rate between the two bands in the presence of an electric field by considering WSL. The transition probability rate, thus obtained, contains a steady state term independent of WSL and an oscillatory term due to the same. The steady state term never shows oscillation for photon energy both below and above E_g. Besides, the oscillations, that are obtained, are due to consideration of WSL. Furthermore, Callaway neglected the dependence of the OME on the $\bar{k}$ in evaluating the interband transition matrix element [11]. Incidentally, the OME cannot be taken constant with respect to $\bar{k}$ in the formulation of the interband transition probability rate [14].

The main purpose of the present study is to re-analyze Callaway's formulation for the interband transition matrix element, considering the fact that the OME depends on the electron wave-vector and to show that for photon energy greater than E_g, the oscillations are initiated in the OAC without any effect of the WSL; where as for photon energies below E_g, no such oscillations occur. The oscillations that are due to Callaway considering the WSL, are the additional ones is our case. Therefore, the effect of a longitudinal magnetic field on the above is studied based on the facts that the transverse wave-vector ($k_\perp$) is conserved in the interband transition process, following the interband tunneling theory in the presence of a parallel magnetic field due to Argyres [10]. Similar results as in the case of electric field are also obtained for this case with the modifications in the expressions of the OAC.

In Section 2 of the theoretical background, we have derived the OAC in the presence of an electric field only. Thereafter, we have proceeded with the steady state term to evaluate the transition probability rates for the purpose of the formulation of the OAC. In Section 3, the effect of a parallel magnetic field on the interband transition probability rate and then the OAC has been calculated. In Section 4, we have discussed our results taking n-GaAs as an example for the purpose of numerical computations.

2. Theoretical Background

2.1. Formulation of the Optical Absorption Coefficient in the presence of an electric field only

The interband optical absorption coefficient, α_o, can be written as

$$\alpha_o(\omega, F, H) = \frac{2\hbar P(\omega, F, H)}{\omega n \varepsilon_o c A_0^2}, \tag{1}$$

where, $\hbar = h/2\pi$, h is the Planck constant, $P(\omega, F, H)$ is the rate of interband transition probability per unit time per unit volume, in presence of an electric field ($f = F/e$, in which F is the electric force and e is the electron charge) and a parallel magnetic field (H). ω is the angular frequency of the incident radiation, n is the refractive index of the semiconductor, ε_o is the permitivity of the free space, c is the velocity of light in free space, and A_o is the amplitude of the incident light wave.

In the presence of a uniform electric field of force, F, applied along the x-direction, the effective Schrdinger equation for one electron can be written as

$$\left[E_l(\bar{k}) - \mathrm{i}F\frac{\partial}{\partial k_x}\right]A_{l,v} = W_{l,v}A_{l,v}, \tag{2}$$

where, $E_l(\bar{k})$ is the energy of a Bloch electron in band l, $\bar{k}$ is the electron wave vector, k_x is the x-component of $\bar{k}$, $A_{l,v}$ is the amplitude of the wave function and $W_{l,v}$ is the eigen-energy value of the Bloch electron. It is assumed that the direction of the electric field coincides with one of the reciprocal lattice vectors of the solids. The solution of Eq. (2) is given by

$$A_{l,v} = \frac{1}{G^{1/2}}\exp\left[\frac{\mathrm{i}}{F}\int_0^{k_x}\left\{W_{l,v} - E_l(k_\perp, k_x')\right\}\mathrm{d}k_x'\right], \tag{3}$$

where,

$$W_{l,v} = \frac{2\pi vF}{G} + \frac{1}{G}\int_{-G/2}^{G/2}E_l(k_\perp, k_x)\,\mathrm{d}k_x, \tag{4}$$

in which $k_\perp$ is the normal component of the wave vector, $\bar{k}$, G is the width of the Brilloin's zone in the x-direction, $i = \sqrt{-1}$, and $v(= 0, \pm1, \pm2, ...)$ is an integer showing the discrete "Stark" level. The effect of the electric field on the motion of the electron is neglected in this calculation, as there are off-diagonal matrix elements of the Hamiltonian between two bands. This off-diagonal matrix element is necessary for the description of the tunneling. In the present case of the optical absorption, the perturbed term in the total Hamiltonian is given by

$$H' = \frac{e}{m}(\bar{A}.\bar{p}), \tag{5}$$

where, e is the electronic charge, m is the free electron mass, $\bar{A}$ is the vector potential due to photon, and $\bar{p}$ is the linear momentum operator. The matrix element of transition