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Electron Raman scattering in quantum well wires

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Abstract

Electron Raman scattering (ERS) is investigated in a semiconductor quantum well wire (QWW) of cylindrical geometry for $T = 0\,\mathrm{K}$ and neglecting phonon-assisted transitions. The differential cross-section (DCS) involved in this process is calculated as a function of a scattering frequency and the cylindrical radius. Electron states are confined within a QWW. Single parabolic conduction and valence bands are assumed. The selection rules are studied. Singularities in the spectra are interpreted for various cylindrical radii. ERS discussed here can provide direct information about the electron band structure of the system.

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Keywords: Raman scattering; Differential cross-section; Quantum well wires

1. Introduction

The low-dimensional semiconductor nanostructures have been widely studied for their extraordinary physical properties in recent years. Quantum well wires (QWWs) system is one of these fields of great interest. In this system, the band gap, the electronic structure, transport properties, third-harmonic generation, exciton and impurity levels and binding energies have been broadly investigated [1–11]. Besides, varieties of quantum phenomena such as low-dimensional electron states (hole states), modified dynamics of carriers in the systems and increased exciton binding energy can be provided by nanometer-scale confinements in semiconductor materials [12–15], and those quantum phenomena might be used in the field of solid-state lasers and optoelectronic devices [16].

It is well-known that Raman spectra can be used to investigate various physical properties of semiconductor nanostructures [17–19]. By considering different polarizations of the incident and emitted radiation, the electronic structure of semiconductor materials and nanostructures can be thoroughly investigated [17–20]. Moreover, Raman spectra is being paid close attention to for its possible applications. Based on resonant Raman scattering, Raman injection laser has been produced [21].

Phonon-assisted Raman scattering has been discussed in many nanostructures [22–27]. Meanwhile, Raman scattering without phonon has also been reported, and rich spectra which are the main characteristic of the ERS have been observed [28–30]. However, the difference between the contributions of electron and hole to Raman scattering is seldom discussed. In this paper, the DCS of Raman scattering due to electron and hole without phonon in a GaAs/AlAs QWW is calculated under various cylindrical radii. The electron is completely confined within the QWW system. Single parabolic bands are also considered.

In the following, firstly, the model and the fundamental theory are described in Section 2. Secondly, the DCS is derived in Section 3. Thirdly, the numerical results and relevant discussions are presented for GaAs/AlAs cylindrical QWWs in Section 4. Finally, concise conclusions are showed in Section 5.

2. Model and theory

In this paper, the QWW geometry is cylindrical with circular cross-section of radius r_0 and length L. Consider-

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ing a single conduction (valence) band split into a subband system due to electron confinement within the structure, in the envelope function approximation, the wave functions can be written as [31]

$$\psi_j = [\sqrt{2\pi L}]^{-1} e^{-i(n_j\phi + k_{z_j} z)} u_j \begin{cases} A J_{n_j}\left(\chi_{n_j m_j} \dfrac{r}{r_0}\right), & r \leqslant r_0, \\ B K_{n_j}\left(y_{n_j m_j} \dfrac{r}{r_0}\right), & r > r_0 \end{cases} \quad (1)$$

with

$$B = \frac{J_{n_j}(\chi_{n_j m_j})}{K_{n_j}(y_{n_j m_j})} A$$

and

$$A = \left\{ |J_{n_j}(\chi_{n_j m_j})|^2 + \left[\frac{\mu_2 \chi_{n_j m_j} J'_{n_j}(\chi_{n_j m_j})}{\mu_1 y_{n_j m_j} K'_{n_j}(y_{n_j m_j})}\right]^2 |K_{n_j}(y_{n_j m_j})|^2 \right\}^{-1/2}$$

$$y_{n_j m_j} = \sqrt{\frac{2\mu_2 r_0^2 V_0}{\hbar^2} - \frac{\mu_2}{\mu_1}\chi^2_{n_j m_j}},$$

where $j = 1(2)$ represents the electron (hole). V_0 stands for the band offset, $J_{n_j}(x)$ and $K_{n_j}(x)$ are the Bessel and modified Bessel functions of order n_j , $J'_{n_j}(x)$ and $K'_{n_j}(x)$ are their corresponding partial derivatives. μ_1 and μ_2 are the effective electron masses within the interior and exterior media, respectively. The states are described by the quantum numbers: $n_j = 0, 1, \ldots$; $m_j = 1, 2, \ldots$. k_{z_j} is the wave number of the free motion of electron along the wire axis. u_j is the Bloch function taken at $\mathbf{k}_0 = 0$, where (by assumption) the band extremum is located. On the other hand, the energy levels are determined by

$$E_j = \frac{\hbar^2}{2\mu_j}\left[k_{z_j}^2 + \left(\frac{\chi_{n_j m_j}}{r_0}\right)^2\right]. \quad (2)$$

To determine the total energies, we have to obtain the solution of the secular equation

$$\mu_1 y_{n_j m_j} K'_{n_j}(y_{n_j m_j}) J_{n_j}(\chi_{n_j m_j}) = \mu_2 \chi_{n_j m_j} J'_{n_j}(\chi_{n_j m_j}) K_{n_j}(y_{n_j m_j}).$$

3. Differential cross-section

The DCS per unit solid angle $d\Omega$ of ERS of volume V for incoming light of frequency ω_l and scattering light of frequency ω_s is given by Ref. [31]

$$\frac{d^2\sigma}{d\omega_s\, d\Omega} = \frac{V^2 \omega_s^2 \eta(\omega_s)}{8\pi^3 c^4 \eta(\omega_l)} W(\omega_s, \mathbf{e}_s), \quad (3)$$

where $\eta(\omega)$ is the refractive index as a function of the radiation frequency, $\mathbf{e}_s$ is the polarization vector of the emitted secondary radiation, c is the velocity of light in vacuum and $W(\omega_s, \mathbf{e}_s)$ is the transition rate calculated according to

$$W(\omega_s, \mathbf{e}_s) = \frac{2\pi}{\hbar}\sum_f |M_1 + M_2|^2 \delta(E_f - E_i), \quad (4)$$

where

$$M_j = \sum_a \frac{\langle f|\hat{H}_{js}|a\rangle\langle a|\hat{H}_{jl}|i\rangle}{E_i - E_a + i\Gamma_a} + \sum_b \frac{\langle f|\hat{H}_{jl}|b\rangle\langle b|\hat{H}_{js}|i\rangle}{E_i - E_b + i\Gamma_b}. \quad (5)$$

In Eq. (5), $|i\rangle$ and $|f\rangle$ denote the initial and final states of the system, their corresponding energies are E_i and E_f. $|a\rangle$ and $|b\rangle$ are intermediate states with energies E_a and E_b while Γ_a and Γ_b are the corresponding lifetime widths. In Eqs. (5), the Hamiltonian $\hat{H}_{jl}$ is

$$\hat{H}_{jl} = \frac{|e|}{\mu_0}\sqrt{\frac{2\pi\hbar}{V\omega_l}}\mathbf{e}_l \cdot \hat{\mathbf{p}}, \quad \hat{\mathbf{p}} = -i\hbar\nabla, \ j = 1, 2, \quad (6)$$

where μ_0 is the free-electron mass. This Hamiltonian describes the interaction between electron and the incident radiation field in the dipole approximation. The interaction between electron and the secondary-radiation field is described by

$$\hat{H}_{js} = \frac{|e|}{\mu_j}\sqrt{\frac{2\pi\hbar}{V\omega_s}}\mathbf{e}_s \cdot \hat{\mathbf{p}}, \quad j = 1, 2. \quad (7)$$

This Hamiltonian describes the photon emitted by the electron (hole) when transitions between conduction (valence) subbands occur.

The initial state $|i\rangle$ stands for a completely occupied valence band, an unoccupied conduction band, and an incident photon of energy $\hbar\omega_l$. Thus,

$$E_i = \hbar\omega_l. \quad (8)$$

The final state $|f\rangle$ contains an EHP excited in a real state, a secondary-radiation emitted photon of energy $\hbar\omega_s$. Hence,

$$E_f = E_{n_1 m_1} + E_{z_1} + E_{n_2 m_2} + E_{z_2} + \hbar\omega_s + E_g. \quad (9)$$

For the intermediate states $|a\rangle$ and $|b\rangle$, the energies E_a and E_b can be easily obtained from the above discussion. Using energy and momentum conservation laws, we can calculate the denominators in Eq. (5)

$$E_i - E_a = E_{n_1 m_1} - E_{n'_1 m'_1} + \hbar\omega_s, \quad (10)$$

$$E_i - E_b = E_{n'_2 m'_2} - E_{n_2 m_2} - \hbar\omega_s. \quad (11)$$

Now, we can calculate the matrix elements. If we consider allowed electron transitions between conduction and valence bands, Eqs. (1) and (6), the matrix element $\langle a|\hat{H}_{jl}|i\rangle$, in the envelope function approximation, can be written as

$$\langle a|\hat{H}_{jl}|i\rangle = \frac{i\mu_1|e|}{\mu_0}\sqrt{\frac{2\pi}{\hbar V\omega_l}} W_{n'_1 n_2}\left[\int_0^{r_0} A' J_{n'_1}\left(\chi_{n'_1 m'_1}\frac{r}{r_0}\right) \times \mathbf{e}_l \cdot \mathbf{r} A_2 J_{n_2}\left(\chi_{n_2 m_2}\frac{r}{r_0}\right) r\, dr + \int_{r_0}^{r} B' K_{n'_1}\left(y_{n'_1 m'_1}\frac{r}{r_0}\right) \times \mathbf{e}_l \cdot \mathbf{r} B_2 K_{n_2}\left(y_{n_2 m_2}\frac{r}{r_0}\right) r\, dr\right] \delta_{n'_1, n_2}\delta_{k_{z_a}, k_{z_i}}. \quad (12)$$