Available online at www.sciencedirect.com
ScienceDirect
Physica B 392 (2007) 369–378
PHYSICA B
www.elsevier.com/locate/physb

Study on band gaps of elastic waves propagating in one-dimensional disordered phononic crystals

A.-Li Chen, Yue-Sheng Wang*

Institute of Engineering Mechanics, Beijing Jiaotong University, Beijing 100044, PR China

Received 7 September 2006; accepted 6 December 2006

Abstract

Band gaps of elastic waves, both in-plane and anti-plane waves, propagating along arbitrary direction in one-dimensional disordered phononic crystals are studied in this paper. The localization of wave propagation due to random disorder is discussed by introducing the concept of the localization factor. As a special case between ordered and disordered structures, we analyze the properties of the band gaps of phononic crystals with quasi-periodicity (i.e. phononic quasicrystals). Compared with the periodic structure, phononic quasicrystals involve more bands with localization of wave motion. The transmission coefficients are also calculated and the results show the same behaviors as the localization factor does. Therefore, the localization factor may act as an accurate and efficient parameter to characterize band structures of both ordered and disordered (including quasi-periodic) phononic crystals.

PACS: 63.20.Pw; 63.50.+x; 63.20.−e; 61.44.Br

Keywords: Phononic crystal; Elastic wave; Band gap; Disorder; Quasi-periodicity; Localization; Transfer matrix

1. Introduction

Since Kushwaha [1] proposed the concept of "phononic crystal", an artificial periodic elastic/acoustic structure that exhibits so-called "phononic band gaps" [2], a great deal of attention was focused on this kind of artificial lattice structures [3]. Band gaps involved in phononic crystals have numerous potential engineering applications such as acoustic filters, control of vibration isolation, noise suppression and design of new transducers. So far, several methods were developed to calculate the band gaps of the phononic crystals, for instance, the transfer matrix method [4], plane-wave-expansion method (PWE) [5–7], finite-different time-domain method (FDTD) [8–10], multi-scattering theory (MST) [11] among others [12,13]. A lot of results based on these methods were reported about the band gaps of the perfectly ordered phononic crystals or those with defects [14–16]. However in practical cases, disorder, usually caused by randomly distributed material defaults or manufacture errors during production process, is very common. This may lead to localization phenomenon, like the well-known Anderson localization of electron waves in disordered systems [17].

Since the pioneer works of Anderson [17], localization phenomenon in randomly disordered systems has attracted considerable attention, e.g. localization of acoustic waves (AW) and electromagnetic waves (EW) in media with properties fluctuating randomly [18,19], and vibration localization of nearly periodic engineering structures such as beams, bars, plates, etc. [20,21]. In the recent decade, the band structures and localization phenomenon of EM waves in disordered photonic crystals have also been studied [22–24]. However, researches on randomly disordered phononic crystals are very limited. This topic, we believe, is of practical importance not only because the randomly distributed manufacture errors may cause the disorder as we mentioned before, but also because one may expect to tune the band structures of phononic crystals and thus control the propagation behavior of elastic waves by intentionally introducing disorder. In this paper, the band gaps of elastic waves in one-dimensional (1D) random

*Corresponding author. Tel.: +86 10 51688417; fax: +86 10 51682094.
E-mail address: yswang@center.njtu.edu.cn (Y.-S. Wang).

0921-4526/$ - see front matter
doi:10.1016/j.physb.2006.12.004

disordered phononic crystals are studied. The general case of wave propagation in arbitrary direction will be considered. Transfer matrix method will be employed. Instead of calculating the transmitted waves, we will use a well-defined localization factor [25] to characterize the band structures and localization phenomenon of the system. As expected, the numerical results show that the localization factor predicts the same band structures as the amplitude of the transmitted wave does for both ordered and disordered phononic crystals. In addition, we will also study elastic wave motion in 1D phononic crystals with quasi-periodic structures (Fibonacci sequence). As we know a quasi-periodic system is of the case between ordered and disordered systems, and thus the waves therein have both propagating and localizing modes [26]. Here, we will examine its band structures using the localization factor.

2. Problem statement and solution of wave motion equations

Consider a 1D phononic crystal shown in Fig. 1. The phononic crystal consists of n unit cells. Each unit cell includes two sub-cells made by two different materials (A and B) and denoted by subscript $j = 1, 2$. The thickness, Lamé constant, shearing modulus, Young's modulus and mass density of the sub-cells are denoted by a_j, λ_j, μ_j, $E_j[E_j = \mu_j(3\lambda_j + 2\mu_j)/(\lambda_j + \mu_j)]$ and ρ_j, respectively. So the thickness of a unit cell is $a = a_1 + a_2$. For the present two-dimensional problem, we introduce two displacement potentials, φ and ψ [27], such that the displacement components, v_x and v_y, are written as $v_x = \partial\varphi/\partial x + \partial\psi/\partial y$ and $v_y = \partial\varphi/\partial y - \partial\psi/\partial x$. Then, we have the governing equations for wave motion,

$$\nabla^2\varphi = c_{\mathrm{L}}^{-2}\ddot{\varphi}, \quad \nabla^2\psi = c_{\mathrm{T}}^{-2}\ddot{\psi}, \tag{1}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$; and $c_{\mathrm{L}} = \sqrt{(\lambda + 2\mu)/\rho}$ and $c_{\mathrm{T}} = \sqrt{\mu/\rho}$ are the longitudinal and sheer wave speeds, respectively. It will be convenient to cast the equations into dimensionless forms by introducing the following dimensionless local coordinates:

$$\xi_j = x_j/\bar{a}_1, \quad \eta_j = y_j/\bar{a}_1, \tag{2}$$

where $\bar{a}_1$ is the mean value of the thickness of material A (it is exactly equal to a_1 for the perfectly periodic structure). We consider plane waves propagating in an arbitrary

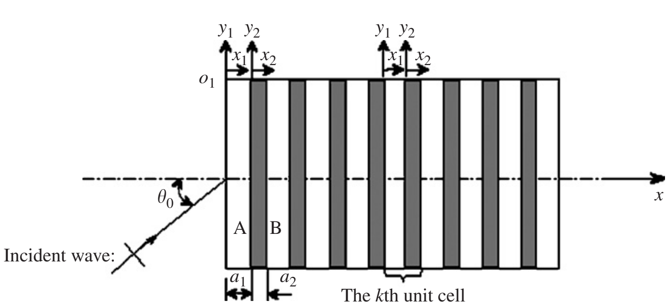

Fig. 1. A schematic diagram of a 1D phononic crystal.

direction with the y-component of the dimensionless wave vector k_y. Therefore, the general dimensionless solutions to Eq. (1) have the forms

$$\begin{aligned}\varphi_j(\xi_j, \eta_j, t) = [&A_1 \exp(-\mathrm{i}q_{\mathrm{L}j}\xi_j) \\ &+ A_2 \exp(\mathrm{i}q_{\mathrm{L}j}\xi_j)] \exp(\mathrm{i}k_y\eta_j - \mathrm{i}\omega t), \\ \psi_j(\xi_j, \eta_j, t) = [&B_1 \exp(-\mathrm{i}q_{\mathrm{T}j}\xi_j) \\ &+ B_2 \exp(\mathrm{i}q_{\mathrm{T}j}\xi_j)] \exp(\mathrm{i}k_y\eta_j - \mathrm{i}\omega t),\end{aligned} \tag{3}$$

where $0 \leqslant \xi_j \leqslant \zeta_j = a_j/\bar{a}_1$; $\mathrm{i}^2 = -1$; $q_{\mathrm{L}j} = \sqrt{(\omega\bar{a}_1/c_{\mathrm{L}j})^2 - k_y^2}$ and $q_{\mathrm{T}j} = \sqrt{(\omega\bar{a}_1/c_{\mathrm{T}j})^2 - k_y^2}$; ω is the circular frequency; A_1, A_2, B_1 and B_2 are the unknown coefficients to be determined. If we consider an incident wave with the phase velocity of c_0 propagates in an arbitrary direction of θ_0 (see Fig. 1), then $k_y = k_0\bar{a}_1 \sin\theta_0$ with $k_0 = \omega/c_0$.

The dimensionless displacement and stress components are given by

$$\begin{aligned}\bar{v}_x &= \frac{\partial\varphi}{\partial\xi} + \frac{\partial\psi}{\partial\eta}, \\ \bar{v}_y &= \frac{\partial\varphi}{\partial\eta} - \frac{\partial\psi}{\partial\xi}, \\ \bar{\sigma}_x &= \lambda\left(\frac{\partial^2\varphi}{\partial\xi^2} + \frac{\partial^2\varphi}{\partial\eta^2}\right) + 2\mu\left(\frac{\partial^2\varphi}{\partial\xi^2} + \frac{\partial^2\psi}{\partial\xi\partial\eta}\right), \\ \bar{\tau}_{yx} &= \mu\left(2\frac{\partial^2\varphi}{\partial\xi\partial\eta} + \frac{\partial^2\psi}{\partial\eta^2} - \frac{\partial^2\psi}{\partial\xi^2}\right),\end{aligned} \tag{4}$$

which will form a state vector in the following analysis.

3. Transfer matrix

We take the dimensionless state vectors at the left and right sides of each sub-cells in the kth unit cell ($k = 1, 2, \ldots, n$) as $\mathbf{V}_{j\mathrm{L}}^{(k)} = \left\{\bar{\sigma}_{xj\mathrm{L}}^{(k)}, \bar{\tau}_{yxj\mathrm{L}}^{(k)}, \bar{v}_{xj\mathrm{L}}^{(k)}, \bar{v}_{yj\mathrm{L}}^{(k)}\right\}^{\mathrm{T}}$ and $\mathbf{V}_{j\mathrm{R}}^{(k)} = \left\{\bar{\sigma}_{xj\mathrm{R}}^{(k)}, \bar{\tau}_{yxj\mathrm{R}}^{(k)}, \bar{v}_{xj\mathrm{R}}^{(k)}, \bar{v}_{yj\mathrm{R}}^{(k)}\right\}^{\mathrm{T}}$, where the subscripts L and R denote the left and right sides of the sub cells. These two state vectors have the following relation:

$$\mathbf{V}_{j\mathrm{R}}^{(k)} = \mathbf{T}'_j\mathbf{V}_{j\mathrm{L}}^{(k)}, \tag{5}$$

where $\mathbf{T}'_j$ are 4×4 transfer matrices whose elements are given in Appendix A, Eq. (A.1). The continuous conditions at the interfaces between the two sub-cells and between the two unit cells lead to the relationship between the state vectors of the (k-1)th and kth unit cells,

$$\mathbf{V}_{2\mathrm{R}}^{(k)} = \mathrm{T}_k\mathbf{V}_{2\mathrm{R}}^{(k-1)}, \tag{6}$$

where $\mathbf{T}_k$ is the transfer matrix between two consecutive unit cells and has the form

$$\mathrm{T}_k = \mathrm{T}'_2\mathrm{T}'_1. \tag{7}$$

It is noted that $\psi = 0$ i.e. $B_1 = B_2 = 0$ for the normal incident case, $\theta_0 = 0^\circ$ ($k_y = 0$). Then in this case, the vectors at the left and right sides of each sub-cells are