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Physica B 373 (2006) 240–244
PHYSICA B
www.elsevier.com/locate/physb

Pseudogap phenomenon and superconductivity in a magnetic field

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Received 16 November 2005; accepted 17 November 2005

Abstract

We study the effect of a magnetic field on the d-density wave (DDW) in the presence of $d_{x^2-y^2}$ superconducting order parameter (DSC) for high-temperature cuprates using a mean-field calculation of the tight binding model. The phase diagrams for the order parameter with doping are discussed. The phase diagram of the field with filling and also the dependence of the critical field (H_c) on the critical temperature (T_c) have been considered. The temperature dependence of the specific heat is also demonstrated. The effect of the field on the DDW order parameter and the superconducting gap are not alike. While the field suppresses the DSC gap, it tends to increase the DDW gap at the same doping. Moreover the gap to T_c ratio increases with the field. The critical field has a power law dependence (H_c–T_c^2).

PACS: 71.10.Fd; 74.72.−h; 71.27.+a; 71.10.Pm

Keywords: High T_c compounds; Magnetic field; Pseudogap

The formation of the pseudogap below a characteristic temperature, T^* is, by now, a well-established concept where the density-of-states is depleted in the underdoped phase of the high-T_c cuprates. Various experiments, such as photoemission [1], specific heat [2], tunneling [3] and optical conductivity [4] have observed the pseudogap in cuprates. However, the origin of the pseudogap remains controversial and has been ascribed to two different scenarios, occurrence of a competing order parameter or to the presence of the precursor superconductivity. The magnetic effects are good candidates for distinguishing between the two scenarios. The pseudogap in the high-temperature superconductors has recently been considered as a d-density wave order (DDW) [5,6] where the particle hole pair of the same orientation form the condensate. The phase diagram of the cuprates are in agreement with this recent picture. Moreover, the experimental situation is now quite promising. The elastic neutron scattering [7] and various other experiments [8] are consistent with the above picture. The pseudogap exhibits a $d_{x^2-y^2}$-wave symmetry alike the superconducting gap below T_c. However, the broken symmetry of the DDW state makes it different from the $d_{x^2-y^2}$-wave superconducting gap of the Cooper pairs.

The magnetic field applied perpendicular to the CuO_2 planes has been a very efficient instrument to understand both the normal and superconducting phases of the cuprates. The magnetic field study of the superconductors in presence of the spin-density wave (SDW) has shown that the SDW order grows with magnetic field wheras the superconductivity is suppressed [9,10]. Most importantly, the field dependence of the SDW Bragg peak intensity has a cusp at the zero field. There are quite a few experiments that shed light on the effect of the magnetic field on the pseudogap. Nuclear magnetic resonance experiments on underdoped $YBa_2Cu_4O_8$ [11] exhibit a weak dependence of the pseudogap on an applied perpendicular magnetic field. Experiments on nearly optimally doped $YBa_2Cu_3O_{7-\delta}$ [12] share the same view. Moreover, a systematic determination of the pseudogap closing field has been made applying magnetic fields up to 60 T [13]. The above field and the temperature T^* are related through the simple Zeeman-like expression that suggests the coupling of the magnetic field to the pseudogap by the Zeeman energy of the spin degrees

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doi:10.1016/j.physb.2005.11.150

of freedom, thus entailing a predominant role of the spins over the orbital effects in the formation of the pseudogap.

Inspite of the various experimental facts and theoretical works, the question of the nature and origin of the pseudogap in the normal phase is still open. The pseudogap in our model has been considered to be the DDW order. The DDW order has successfully explained various properties of the pseudogap phase in the normal state of the high T_c materials. However, there is a sea of confusion with respect to the effect of the magnetic field on the pseudogap in presence of the superconductivity. Thus, it is of utmost importance to study the effect of the magnetic field on DDW. The magnetic field dependence of the DDW order has recently been studied in Ref. [14]. They have also observed the de Hass–van Alphen oscillations in a DDW state unlike the d-wave superconducting state as is evident from the difference in the behavior of the nodal quasi-particles of the two states. In this work, we consider the magnetic field dependence of the d-density wave order parameter and the $d_{x^2-y^2}$ superconducting order parameter (DSC) in a tight binding model. The applied magnetic field in this case considers the coupling of the magnetic field to the quasi-particle spin and modifies the hopping term. It is important to study the effect of the magnetic field involving the spin term as well as the orbital terms. In this work we consider the magnetic field involving only the spin degrees of freedom. Later we would like to include the orbital effects to see how it changes. As the paramagnetism is an observable phenomenon it justifies our study. Similar works based on the other scenarios of pseudogap like the precursor superconductivity including coupling of the pseudogap by the spin degrees of freedom have been made [15].

We consider a two-dimensional tight binding model with appropriate lattice symmetry and second nearest neighbor hopping parameter. A mean-field calculation is performed to obtain the phase diagrams with doping, δ and also the temperature dependence of specific heat. The dependence of the critical field on doping and critical temperature are also studied. The understanding of the effect of the magnetic field on the three types of states, namely the DDW, DDW + DSC and DSC, is important for the understanding of the superconducting phase and the normal phase in the underdoped region

The Hamiltonian can be written as

$$H = \sum_{\mathbf{k}\sigma}(\varepsilon_{\mathbf{k}} - \mu)c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\sigma}\mathrm{i}W_{\mathbf{k}}c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}+\mathbf{Q}\sigma} + \mathrm{h.c} + \sum_{\mathbf{k}}\Delta_{\mathbf{k}}c^\dagger_{\mathbf{k}\uparrow}c^\dagger_{-\mathbf{k}\downarrow} + \mathrm{h.c} + \mu_B\sum_{i\sigma\sigma'}c^\dagger_{i\sigma}(\boldsymbol{\sigma}_{\sigma\sigma'}.\mathbf{H})c_{i\sigma'}, \tag{1}$$

where $c^\dagger_{\mathbf{k}\sigma}$ is the fermion creation operator for wavevector $\mathbf{k}$ and spin σ. $W_{\mathbf{k}}$ and $\Delta_{\mathbf{k}}$ are the DDW gap and the DSC gap, respectively. $\mathbf{Q}$ is the nesting vector (π,π) and $\mathbf{H}$ is the external magnetic field applied perpendicular to the cuprate planes. μ is the chemical potential. In our model with nearest neighbor hopping, we have $\mu = 0$ at half filling, while it takes negative values away from half filling. $\boldsymbol{\sigma}_{\sigma\sigma'}$ are the components of the Pauli matrices: $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$. The d-density wave and the superconducting gaps are given by

$$W_{\mathbf{k}} = V_{\mathrm{DDW}}\phi_{\mathbf{k}}\sum_{\mathbf{k}'}\langle c^\dagger_{\mathbf{k}'\uparrow}c_{\mathbf{k}'+\mathbf{Q}\uparrow} + c^\dagger_{\mathbf{k}'\downarrow}c_{\mathbf{k}'+\mathbf{Q}\downarrow}\rangle,$$
$$\Delta_{\mathbf{k}} = V_{\mathrm{DSC}}\eta_{\mathbf{k}}\sum_{\mathbf{l}}\langle c_{-\mathbf{l}\uparrow}c_{\mathbf{l}\downarrow}\rangle, \tag{2}$$

where V_{DDW} is the pairing interaction for the density wave channel. $\phi_{\mathbf{k}}$ represents the basis function which determines the symmetry of the order parameter and the corresponding pairing interaction; $\phi_{\mathbf{k}} = \cos k_x - \beta\cos k_y$. Here $\beta = 1$ corresponds to square lattice and $\beta \neq 1$ represents orthorhombic distortion. V_{DSC} is the pairing interaction and $\eta_{\mathbf{k}}$ is the basis function that determines the symmetry of the superconducting order parameter. The energy dispersion relation is given by $\varepsilon_{\mathbf{k}} = -2t(\cos k_x + \beta\cos k_y - 2\gamma\cos k_x\cos k_y)$ which gives the band structure. t and βt are the nearest neighbor hopping integrals along the in-plane a and b axes respectively. The model is more realistic due to the next-nearest-neighbor hopping γt.

The mean-field Hamiltonian of the system in the Nambu basis can be written as

$$H_{\mathrm{MF}} = \sum \psi^\dagger_{\mathbf{k}}M_{\mathbf{k}}\psi_{\mathbf{k}} + E_0, \tag{3}$$

where $\psi^\dagger_{\mathbf{k}} = (c^\dagger_{\mathbf{k}\uparrow}, c_{-\mathbf{k}\downarrow}, c^\dagger_{\mathbf{k}+\mathbf{Q}\uparrow}, c_{-\mathbf{k}-\mathbf{Q}\downarrow})$. E_0 is the constant energy shift. $M_{\mathbf{k}}$ is given by

$$M_{\mathbf{k}} = \begin{pmatrix} \varepsilon_{\mathbf{k}} - \mu + \mu_B H & \Delta_{\mathbf{k}} & \mathrm{i}W_{\mathbf{k}} & 0 \\ \Delta_{\mathbf{k}} & -(\varepsilon_{\mathbf{k}} - \mu) + \mu_B H & 0 & \mathrm{i}W_{\mathbf{k}} \\ -\mathrm{i}W_{\mathbf{k}} & 0 & -(\varepsilon_{\mathbf{k}} + \mu) + \mu_B H & -\Delta_{\mathbf{k}} \\ 0 & -\mathrm{i}W_{\mathbf{k}} & -\Delta_{\mathbf{k}} & (\varepsilon_{\mathbf{k}} + \mu) + \mu_B H \end{pmatrix}. \tag{4}$$

The d-density wave order parameter is purely that results in a broken symmetry state with a finite spin current. The above matrix $M_{\mathbf{k}}$ has four eigenvalues, $E_\nu(\mathbf{k})$ given by

$$E_\nu(\mathbf{k}) = \mu_B H \pm \left[\varepsilon^2_{\mathbf{k}} + \mu^2 + \Delta^2_{\mathbf{k}} + W^2_{\mathbf{k}} \pm 2\sqrt{\varepsilon^2_{\mathbf{k}}\mu^2 + \mu^2 W^2_{\mathbf{k}}}\right]^{1/2}. \tag{5}$$

In short notation we can write $E_\nu(\mathbf{k}) = \mu_B H + s_3 g_{s_2}(\mathbf{k})$ where s_2 and s_3 can take values of ± 1. The magnetic field has been considered to be perpendicular to the c-axis. The inclusion of the magnetic field along the planes will increase the computational difficulties due to increase of the dimension of the matrix. The self consistent equations for DDW in the presence of the d-wave superconductivity can be obtained from the free energy expression given by the trace formula. The two mean-field equations obtained from