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"Moving" Prandtl–Ishilinskii operators with compensator in a closed form

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Abstract

The present paper proposes a class of Preisach-like hysteresis models with saturation having the compensator operator in a closed form, where a mean field contribution (moving model or feedback model) is also considered. The paper pays attention to the conditions for which such a "moving" model is well-defined. Moreover, attention is given to the identification procedures requiring a limited amount of data, where some constraints enhance the correspondence of the model to the underlying hysteresis. Examples showing performances of the model in cases of physical interest are provided.

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1. Introduction

The necessity to describe hysteresis phenomena in several branches of physics and technology has brought to increase investigations in mathematical models of hysteresis, by which several hysteresis phenomena can be described in a phenomenological manner. In such a framework and in particular in applications of control of smart devices employing magneto-elastic materials, the concept of compensators, or inverses, of hysteresis models, [1,2] has widely been investigated in order to allow a *model-based* control. Among several approaches those defining hysteresis operators with the compensator in a closed form [3–5] have turned out very interesting for the need of a limited set of measured data, limited computational cost and acceptable accuracy. A former paper presented a class of Preisach models with a compensator in a closed form [5]; the present paper further enlarges this class of models by adding a mean field contribution, which is widely applied in several tasks, [6]. While results in Ref. [5] were encouraging, some inaccuracy in describing the underlying hysteresis phenomena was detected. These were due to the identification procedure where some important *qualitative* properties of the underlying hysteresis phenomena were not constrained.

The present paper elucidates the existence criteria of the extended class of hysteresis operators. Furthermore, it defines, on one hand, a more general identification procedure where suitable physical constraints are assumed, and addresses, on the other hand, a procedure to identify the unknowns of the feedback model in the case of a special but actual distribution function.

2. Definition of hysteresis model

We consider the following class of hysteresis operators:

$$y = af(x) + \int_0^{+\infty} Q(u)\mathscr{P}_u f(x)\,\mathrm{d}u, \tag{1}$$

where $Q(u) \geqslant 0$ and $f(x)$ is strictly increasing and odd. Such an equation defines a class of Preisach operators with distribution function $\mu(\alpha,\beta) = \frac{1}{4}Q((f(\alpha)-f(\beta))/2)f'(\alpha)f'(\beta)$. This model employs the formalism proposed in Ref. [4] where the first term can be taken into account by

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assuming $\hat{Q}(u) = Q(u) + a\delta(u)$, with δ being the Dirac function. The model described so far generalizes the class of Preisach operators (see Ref. [3]), having the inverse in a closed form

$$x = f^{-1}\left(\frac{1}{a}y + \int_0^{+\infty} (\varphi^{-1})''(u)\mathscr{P}_u y\,\mathrm{d}u\right), \tag{2}$$

where

$$\varphi(r) = ar + \int_0^r Q(u)(r-u)\,\mathrm{d}u. \tag{3}$$

The φ-function defined in Eq. (3) is related to the Everett function as $E(\alpha,\beta) = \varphi[(f(\alpha) - f(\beta))/2]$. Moreover, Eq. (2) shows also that the inverse model can be handled with the same "machinery" of the Preisach operator.

The model defined in Eq. (1) can be further generalized by introducing a *mean field* contribution (or nonlinear feedback) $g(y)$. The equation

$$y = af(x + g(y)) + \int_0^{+\infty} Q(u)\mathscr{P}_u f(x + g(y))\,\mathrm{d}u \tag{4}$$

so defines a model of hysteresis with feedback, provided that specific conditions are fulfilled. To this aim, we preliminary consider a linear feedback $g(x) = Kx$ in a heuristic manner, cf. Ref. [7]. In this case, the model admits only one solution as long as $1/K > \chi_{\max}$, where $\chi_{\max}$ is the maximum susceptibility. Under these conditions, the model with feedback also fulfils the wiping out property and, therefore, is a Preisach-like operator.

In the case of a *nonlinear* feedback, the model (4) is equivalent to the following:

$$y = af(z) + \int_0^{+\infty} Q(u)\mathscr{P}_u f(z)\,\mathrm{d}u, \tag{5}$$

$$y = G(z - x) \tag{6}$$

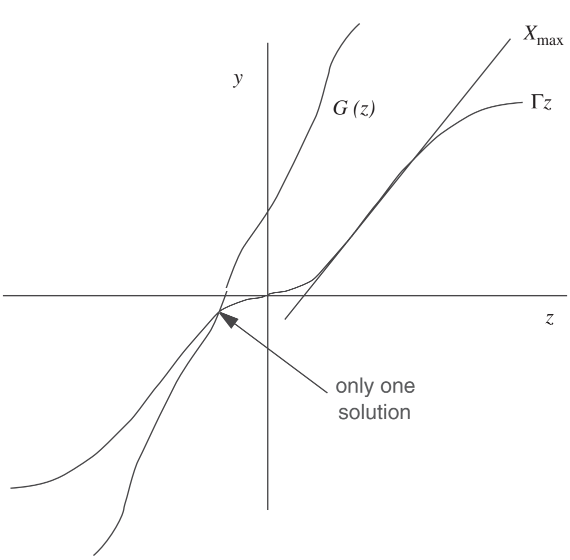

Fig. 1. Solution of the implicit equation defining the model with nonlinear feedback.

with $G = g^{-1}$. It has a unique solution, provided that g is odd and strictly monotone, and that the sufficient condition

$$\frac{\mathrm{d}G}{\mathrm{d}z} \geqslant \frac{1}{K} > \chi_{\max} \quad \text{or} \quad \frac{\mathrm{d}G}{\mathrm{d}z} \leqslant 0 \tag{7}$$

holds for every $z \in \mathbb{R}$, as shown in Fig. 1. Under such conditions, the model's inverse is well defined and has the following closed form

$$x = -g(y) + f^{-1}\left(\frac{1}{a}y + \int_0^{+\infty} (\varphi^{-1})''(u)\mathscr{P}_u y\,\mathrm{d}u\right). \tag{8}$$

Moreover, exploiting the same arguments as in the linear case, we can conclude that the model fulfils the wiping-out property.

3. Identification procedure

The model (4) requires a suitable identification procedure, in order to determine the unknown functions φ, f and g. To this aim one may consider data picked up on the anhysteretic curve $(x_\mathrm{A}, y_\mathrm{A})$, on the descending branch of the limiting loop $(x_\mathrm{d}, y_\mathrm{d})$ and on the virgin curve $(x_\mathrm{v}, y_\mathrm{d})$, respectively:

$$x_\mathrm{A} + g(y_\mathrm{A}) = F(y_\mathrm{A}), \tag{9}$$

$$x_\mathrm{d} + g(y_\mathrm{d}) = F\left[f_\infty - 2\psi\left(\frac{y_\infty - y_\mathrm{d}}{2}\right)\right], \tag{10}$$

$$x_\mathrm{v} + g(y_\mathrm{v}) = F(\psi(y_\mathrm{v})), \tag{11}$$

having assumed $Q \in L^1(\mathbb{R}^+)$, and $\int_{\mathbb{R}^+} Q(u)\,\mathrm{d}u = 1$, and $F = f^{-1}$. Moreover, f_∞ and y_∞ are the saturation values of $f(x)$ and y, respectively, and $\psi = \varphi^{-1}$. The above equations address the identification problem of the model (4). But when available data is limited to e.g. $(x_\mathrm{d}, y_\mathrm{d})$ and $(x_\mathrm{v}, y_\mathrm{d})$, Eq. (9) is not considered at all. As a consequence, we have studied $g = 0$ (no feedback) assuming f- and ψ- functions unknown, as well as $g \neq 0$ (feedback present) considering the relevant case of an assigned Q-function [6], assuming g and F unknown.

In the case $g = 0$, the functions ψ and f may be parameterized by using equidistant triangular base-functions, $\phi_\mathrm{Tr}(x) = 1 - |x|$ when $|x| < 0$ (0 otherwise), giving the piece-wise linear estimates

$$\psi(y) = \sum_{k=-d_\psi}^{d_\psi} a_k \cdot \phi_\mathrm{Tr}(y/T_\psi - k), \tag{12}$$

$$f(x) = \sum_{k=-d_f}^{d_f} b_k \cdot \phi_\mathrm{Tr}(x/T_f - k), \tag{13}$$

where $T_\psi = y_{\max}/d_\psi$ and $T_f = x_{\max}/d_f$ are the respective step-lengths. Estimates of the functions are obtained by minimizing the error between measured data and their corresponding estimates. Having assumed $Q \geqslant 0$ and f