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Spectral density of Cooper pairs in two level quantum dot–superconductors Josephson junction

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ABSTRACT

In the present paper, we report the role of quantum dot energy levels on the electronic spectral density for a two level quantum dot coupled to s-wave superconducting leads. The theoretical arguments in this work are based on the Anderson model so that it necessarily includes dot energies, single particle tunneling and superconducting order parameter for BCS superconductors. The expression for single particle spectral function is obtained by using the Green's function equation of motion technique. On the basis of numerical computation of spectral function of superconducting leads, it has been found that the charge transfer across such junctions can be controlled by the positions and availability of the dot levels.

1. Introduction

Due to recent advancement in nanoscale fabrication techniques, it has become reality to produce hybrid systems of quantum dot. These hybrid structures has quantum dot with well-connected electrodes of ferromagnetic material, metal, and superconductors etc. Out of these combinations, S-QD-S junction, in which a quantum dot sandwiched between two superconducting electrodes, has been widely researched to study the electronic transport across quantum dot [1–8]. The behavior of these S-QD-S junctions is similar to Josephson junctions where the wave function of Cooper pairs leaks across the junction [9]. Superconducting transport through such S-QD-S junction can be described in the light of Andreev reflection mechanism and typical QD phenomena like Coulomb blockade and Kondo effect [10,11].

The electronic transport through the S-QD-S junction owns two processes, one is tunneling of singly charged quasiparticle and the other is Cooper pair transfer between superconducting electrodes without pair breaking effect, which mainly depends on the applied gate voltage across the junction, energy levels in quantum dot and superconducting order parameter of the electrodes [12– 15]. In the absence of gate voltage, the incoming electron from one superconducting lead to quantum dot suffer a Coulombic repulsion due to available electrons in quantum dot and the mechanism is known as Coulomb blockade in literature. The applied gate voltage can tune the Coulomb blockade effect and hence control over the electric charges transfer through the junction. The correlation effects due to the Coulomb blockade and eventually, at sufficient low temperature and zero gate voltage, can induce a sharp resonance peak at the Fermi level significantly enhancing conductance to the unitary value $2\,e^2/h$ [16,17]. For sufficiently high value of Coulomb blockade on dot, the pairing (of opposite spin) is suppressed, and quasiparticle tunnelling is therefore the dominant transport mechanism. When the coherence length of the connected superconductor is greater than the size of the QD junction, there is a possibility of tunneling of Cooper pair from one side of superconductor to other side superconductor without any pair breaking effect and the resulting current is due to the Josephson Cooper pair tunneling in S-QD-S junction [18-21]. This nontrivial competition between the induced pairing and correlations has been widely explored experimentally and theoretically for the hetrostructures compirising the quantum dots hybridised to the superconducting electrodes.

The discrete energy levels present in quantum dot also play a pivotal role in charge transport through nanoscopic junctions and display rich physics to fabricate nano-electronic devices. The discrete energy levels in quantum dot can accommodate electrons and add or remove electron by controlling the gate voltage at the electrodes. This property of quantum dot allows them to be a major part of advanced devices such as single electron transistors. Most of the studies in this area are focused on single level quantum dot to analyze the behavior of Josephson supercurrent between two superconducting leads. These studies concludes that quantum dot level energy, coupling parameter of dot states with superconducting leads controls the charge transport through such

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tunnel junctions. A maximum Josephson supercurrent (known as Resonance Josephson supercurrent) across the S-QD-S junction is reported when the energy of the dot level matches with the Fermi level of the Superconductor-QD-tunnel junction [3-8].

In recent years there has been an increasing interest in more real and complex situations where electronic transport occurs through more than a single dot or where multiple quantum channels in a single dot are involved [22-27]. These configurations could allow the possibility of creating entangled electron pairs and explore nonlocal electronic transport by means of crossed or nonlocal Andreev processes. The availability of multiple energy levels in quantum dot provides a possibility of magnetic coupling of the electron spins localized within the dot and enhance the Kondo temperature by orders of magnitudes. In multilevel quantum dot transport, one can expect a possible competition between the Kondo effect and magnetic coupling of spins localized within the dot. This may open the possibility to control the spin state of the dot system by means of Josephson current and provide a guideline for the fabrication of more advanced nano electronic devices and quantum computer. For a two level dot, the Josephson supercurrent in S-QD-S junction shows remarkable phenomena. There is a chance for Cooper pair to tunnel through the dot and its electrons occupy both the transverse levels of the quantum dot. This phenomenon is again a very useful tool in order to control the Josephson supercurrent through S-QD-S junction.

The existence of multi levels in a quantum dot make the S-QD-S junction device to be very sensitive to interference effects. The asymmetric Fano line shapes occur when electron waves transmitted via a continuum between the SC leads, interfere with the electron waves resonantly scattered by the discrete levels. Sign of this interference can be observed in spectral density of superconducting leads. Because the tunneling of electrons through the junction is greatly affected by the physical properties of the segment between superconducting electrodes, the study of the electronic spectra provides a new way to investigate the electronic properties of the medium Hence, in order to understand the physics of 'multi-level' quantum dots, we have analyzed the spectral function for two level quantum dot coupled to s-wave superconducting leads under the influence of various parameters of connected superconducting leads and number of levels present in the quantum dot for such nanoscopic junctions.

2. Theoretical formulation

The model for two level Quantum dot coupled to s-wave superconducting leads can be described along lines of Anderson model and is given as follows:

$$H = H_D + \sum_{\eta=1,2} (H_\eta + H_{tunn,\eta}) \tag{1}$$

Where,

$$H_{Dot} = \sum_{i=1,2,\sigma} \varepsilon_i d^+_{i\sigma} d_{i\sigma} \tag{1a}$$

$$H_\eta = \sum_{k\sigma} \varepsilon_k a^+_{\eta k\sigma} a_{\eta k\sigma} - \sum_{kk'} (\Delta_{\eta k} a^+_{\eta k'\uparrow} a^+_{\eta -k'\downarrow} + \Delta^+_{\eta k'} a_{\eta -k'\downarrow} a_{\eta k'\uparrow}) \tag{1b}$$

$$H_{tunn,\eta} = \sum_{k\sigma} V(a^+_{\eta k\sigma} d_{1\sigma} + d^+_{1\sigma} a_{\eta k\sigma}) + V(a^+_{\eta k\sigma} d_{2\sigma} + d^+_{2\sigma} a_{\eta k\sigma}) \tag{1c}$$

The Eq. (1a), (1b) and (1c) represents the Hamiltonians for two level QD, effective BCS Hamiltonian for left ($\eta = 1$) and right ($\eta = 2$) side superconductor, and the equal possibility of the single particle tunneling from left superconducting leads through the two levels on the QD and vice-versa respectively. The $a_{\eta k\sigma}$ ($a^+_{\eta k\sigma}$) represents the annihilation (creation) operators for the superconducting lead and d_σ (d^+_σ) represents the annihilation (creation) operators for the dot states.

In order to obtain the expression for electronic spectral density for superconductor, and analyze the competitive role of single particle tunneling and Josephson supercurrent through two level quantum dot of such junction, we have employed the Green's function equation of motion technique [28]. Finally for our model Hamiltonian, we obtain the following Green's function:

$$G_{11}(k,\omega) = \langle\langle a^+_{1-k\downarrow}; a^+_{1k\uparrow} \rangle\rangle = \frac{DB' - D'B}{CB' - BC'} \tag{2}$$

where,

$$B = \frac{(\omega^2 - \varepsilon_k^2 - \Delta^2)}{\Delta} \left\{ 2 - \frac{(\omega - \varepsilon_k)(\omega - \varepsilon_1)(\omega - \varepsilon_2)}{V^2[(\omega - \varepsilon_2) + (\omega - \varepsilon_1)]} \right\}$$

$$B' = -\frac{(\omega + \varepsilon_1)(\omega + \varepsilon_2)(\omega^2 - \varepsilon_k^2 - \Delta^2)}{V^2[(\omega + \varepsilon_2) + (\omega + \varepsilon_1)]}$$

$$C = -\frac{(\omega - \varepsilon_1)(\omega - \varepsilon_2)(\omega^2 - \varepsilon_k^2 - \Delta^2)}{V^2[(\omega - \varepsilon_2) + (\omega - \varepsilon_1)]}$$

$$C' = \frac{(\omega^2 - \varepsilon_k^2 - \Delta^2)}{\Delta} \left\{ 2 - \frac{(\omega + \varepsilon_k)(\omega + \varepsilon_1)(\omega + \varepsilon_2)}{V^2[(\omega + \varepsilon_2) + (\omega + \varepsilon_1)]} \right\}$$

$$D = \frac{(\omega - \varepsilon_k)}{2\pi\Delta} - \frac{(\omega - \varepsilon_1)(\omega - \varepsilon_2)(\omega^2 - \varepsilon_k^2 - \Delta^2)}{V^2[(\omega - \varepsilon_2) + (\omega - \varepsilon_1)]}$$

$$D' = -\frac{1}{\pi}$$

Here, we assume that both superconductors are identical and have same superconducting order parameter (i.e. $\Delta_1(\Delta_1^+) = \Delta_2(\Delta_2^+) = \Delta(\Delta^+)$). Eq. (2) can be rearranged as:

$$G_{11}(k,\omega) = \frac{1}{2\pi} \sum_{i=1}^{8} \frac{I_i}{(\omega - \alpha_i)} \tag{3}$$

where, $\alpha_{1 \ldots\ldots} \alpha_8$ are the eight quasiparticle energies corresponding to eight poles and $I_1 \ldots\ldots I_8$ are the respective weights for each poles of the above Green's function equation. The spectral function A(k,ω) can be calculated from the above Green function $G_{11}(k,\omega)$numerically by using the relationship [28]

$$A(k,\omega) = -\frac{1}{\pi} Im G_{11}(k,\omega), \tag{4}$$

where 'Im' stand for imaginary part of Green's function. Using Eqs. (3) and (4) the expression for electronic Spectral function from $G_{11}(k,\omega)$ can be written as:

$$A(k,\omega) = \frac{1}{\pi^2} \sum_{i=1}^{8} \frac{\Gamma I_i}{\Gamma^2 + (\omega - \alpha_i)^2} \tag{5}$$

In order to match the line shape of the peaks in the ARPES spectral function, the calculations of A(k, ω) would require to be given a broadening of the quasiparticle peaks. Here we have used a Lorentizian type of broadening given by the relationship:

$$\delta(\omega - \tilde{\varepsilon}_k) \cong \frac{1}{\pi} \lim_{\Gamma \to 0} \frac{\Gamma}{(\omega - \tilde{\varepsilon}_k)^2 + \Gamma^2} \tag{6}$$

The broadening parameter Γ is taken to be independent of k and ω [29].

3. Results and discussion

We have analyzed the spectral density A(k, ω) (Eq. 4) for s-wave superconducting leads coupled to a two level quantum dot. A numerical computation of the theoretically calculated A(k,ω) has