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Analysis of cutoff frequency in a one-dimensional superconductor-metamaterial photonic crystal

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ABSTRACT

In this paper, using the two-fluid model and the characteristic matrix method, we investigate the transmission characteristics of the one-dimensional photonic crystal. Our structure composed of the layers of low-temperature superconductor material (*NbN*) and double-negative metamaterial. We target studying the effect of many parameters such as the thickness of the superconductor material, the thickness of the metamaterial layer, and the operating temperature. We show that the cut-off frequency can be tuned efficiently by the operating temperature as well as the thicknesses of the constituent materials.

1. Introduction

Photonic crystals (PCs) extensively demonstrated in many applications due to their inherent physical and optical properties. PCs were obtained theoretically by Yablonovitch [1] and experimentally by John [2]. PCs usually are artificial and composite structures with periodic modulation of refractive index in one, two, and three dimensions. The periods are the order of a fraction of the optical wavelength [3]. PCs are characterized by the appearance of the so-called photonic band gaps (PBGs) in which the propagation of the incident electromagnetic waves prohibited when their frequencies are existing within these PBGs [1,2]. The appearance of these PBGs mainly suspended with the presence of a high dielectric contrast between the consistent materials of PCs, and sometimes called Bragg gaps. Therefore, the small contrasts between the dielectric materials have restricted the set of dielectrics [4].

Meanwhile, the inclusion of the dispersive materials, such as semiconductors [5], metals [6], and superconductors [7–9] in PCs has a significant effect on the PBG modulation. The tunability of PCs can be obtained. Such materials have the tuning advantage due to the tremendous significance of external magnetic field and temperature on their permittivities. However, the superconducting materials have an advantage over the other types of the dispersive materials. At frequencies smaller than the superconducting gap, the scattering of the incident electromagnetic waves due to the imaginary part of the dielectric function is much less than for metallic particles [8]. Therefore, the dielectric losses caused by the superconducting PCs are small as comparable with those of metal PCs.

In 1968, Veslago [10] predicted the existence of a new type of materials in which the refractive index is negative. These materials are called metamaterials or negative index materials (NIMs) with simultaneously negative permittivity and permeability, known as left-handed materials or metamaterials [11]. In addition, their peculiar properties, the experimental investigation of metamaterials at microwave frequencies by Smith et al., [12] devoted the attention toward the usage of metamaterials in the designs and the fabrications of PCs. Moreover, the inclusion of metamaterials in PCs may lead to the appearance of a new band gap with unusual properties over the ordinary PBG [13]. This new gap appears when the average refractive index of the designed PCs equals to zero [13,14]. Therefore, this gap is known as zero-$\bar{n}$ gap.

Based on the novel properties of metamaterials and superconductors, we investigate the optical properties of 1D PCs which is composed of a low-temperature superconductor material (*NbN*) stacked together with a double-negative (DNG) metamaterial. Our calculation method is based on the characteristic matrix method. The numerical results show that the cutoff frequency is strongly affected by the thicknesses of the superconducting and DNG metamaterials. Moreover, the operating temperature can efficiently tune the cutoff frequency. The outlines of this paper are organized as follows. In Section 2, we present 1D PC structure and the basic

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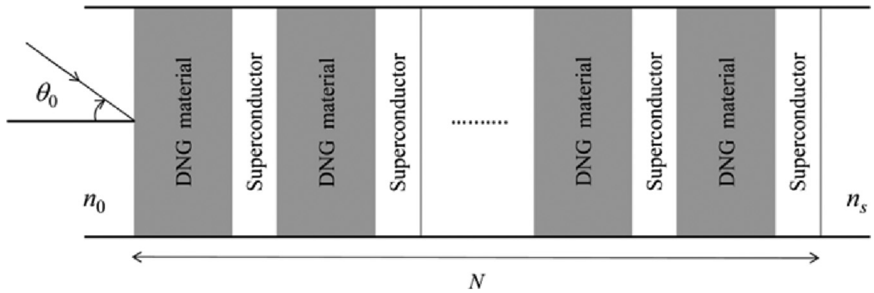

Fig. 1. Schematic diagram of a 1D PCs composed of a superconducting material (*NbN*) and DNG metamaterial layers and repeated for *N* period and the incidence angle is θ_0. Our structure situated between vacuum and substrate of refractive induces n_0 and n_s, respectively.

equations used for our analysis. In Section 3, we present the numerical results and discuss the transmittance spectra of the 1D PCs. The conclusions summarized in Section 4.

2. Model and Basic Equations

Fig. 1 shows the schematic diagram of a 1D PC that composed of a superconductor material (*NbN*) stacked together with a DNG metamaterial for *N* period, which embedded in the air. Here, the dielectric constant of the superconductor material is frequency-dependent and can be described on the basis of the conventional two-fluid model [16]. According to this model, the electromagnetic response of a superconductor can describe in terms of the complex conductivity, $\sigma = \sigma_1 + i\sigma_2$ where, σ_1 and σ_2 indicate to the losses contributed by the normal electrons and the super electrons, respectively. The imaginary part expressed as [16].

$$\sigma_2 = \frac{1}{\omega\,\mu_0\,\lambda_L^2}, \tag{1}$$

where, λ_L is the temperature dependent London penetration depth that given as

$$\lambda_L = \lambda_L(T) = \lambda_0/\sqrt{1-(T/T_c)^4}. \tag{2}$$

Here, λ_0 is the penetration depth at $T=0\,\mathrm{K}$, T is operating temperature, and T_c is the transition temperature of the superconductor. At the critical temperature T_c the superconducting phase vanishes, while, as $T\to 0$, the contribution of the normal electrons is negligible, and the conductivity reduces to

$$\sigma = -i\sigma_2 = \frac{-i}{\omega\,\mu_0\,\lambda_L^2}, \tag{3}$$

Therefore, the dielectric constant can be described as

$$\varepsilon(\omega) = 1 - \frac{c^2}{\omega^2\,\lambda_L^2} = 1 - \frac{\omega_{th}^2}{\omega^2}, \tag{4}$$

where, ω_{th} is the threshold frequency of the superconductor material and c is the velocity of light in a vacuum.

As for the DNG metamaterial layer, the complex permittivity and the permeability of for double-negative metamaterial layer in the microwave region are given by [13,17]:

$$\varepsilon(f) = 1 + \frac{5^2}{0.9^2 - f^2 - i\,\gamma_e\,f} + \frac{10^2}{11.5^2 - f^2 - i\,\gamma_e\,f}, \tag{5}$$

$$\mu(f) = 1 = + \frac{3^2}{0.902^2 - f^2 - i\,\gamma_m\,f}\ ; \tag{6}$$

where, f is the frequency measured in GHz, and γ_e and γ_m are respectively the electric and magnetic damping frequencies that are also given in GHz. Now, we use the previous expressions for the superconductor material and the DNG material on the basis of the characteristic matrix method to describe the interaction of the incident electromagnetic waves within our 1D PCs [15], whereas the single-period characteristic matrix takes the following form,

$$M(a) = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = \begin{pmatrix} \cos\beta_1 & \frac{-i}{p_1}\sin\beta_1 \\ -i\,p_1\sin\beta_1 & \cos\beta_1 \end{pmatrix} \begin{pmatrix} \cos\beta_2 & \frac{-i}{p_2}\sin\beta_2 \\ -i\,p_2\sin\beta_2 & \cos\beta_2 \end{pmatrix}. \tag{7}$$

Where, $(a=d_1+d_2)$ is the lattice constant and the the elements of the matrix M take the following form:

$$m_{11} = \cos\beta_1\,\cos\beta_2 - \tfrac{p_2}{p_1}\sin\beta_1\,\sin\beta_2, \tag{8-a}$$

$$m_{12} = \frac{-i}{p_1}\sin\beta_1\,\cos\beta_2 - \frac{i}{p_2}\cos\beta_1\,\sin\beta_2, \tag{8-b}$$

$$m_{21} = -i\,p_1\sin\beta_1\,\cos\beta_2 - i\,p_2\cos\beta_1\,\sin\beta_2, \tag{8-c}$$

$$m_{22} = \cos\beta_1\,\cos\beta_2 - \frac{p_1}{p_2}\sin\beta_1\,\sin\beta_2, \tag{8-d}$$

where:

$$\beta_1 = \frac{2\pi\,d_1}{\lambda}n_1\,\cos\,\theta_1, \qquad \beta_2 = \frac{2\pi\,d_2}{\lambda}n_2\,\cos\,\theta_2 \tag{9}$$

and, $p_1 = n_1\cos\theta_1$, $p_2 = n_2\cos\theta_2$. For a system of N periods the total characteristic matrix $M(Na)$ of elements M_{11}, M_{12}, M_{21}, and M_{22} can be related to the single period matrix (m_{11}, m_{12}, m_{21},and m_{22}) by the following relations [15]:

$$M_{11} = m_{11}U_{N-1}(\Psi) - U_{N-2}(\Psi), \qquad M_{12} = m_{12}U_{N-1}(\Psi)$$
$$M_{21} = m_{21}U_{N-1}(\Psi) \quad and \quad M_{22} = m_{22}U_{N-1}(\Psi) - U_{N-2}(\Psi) \tag{10}$$

with:

$$\Psi = 0.5\,(m_{11} + m_{22})\ ,\ and \qquad U_N(\Psi) = \frac{\sin\left((N+1)\cos^{-1}\Psi\right)}{\sqrt{1-\Psi^2}} \tag{11}$$

Where, $U_N(\Psi)$ is Chebyshev polynomials of the second kind. Then, the transmission coefficients can be obtained as:

$$t = \frac{2f_0}{(M_{11} + M_{12}\,f_s)f_0 + (M_{21} + M_{22}\,f_s)}, \tag{12}$$

with, $f_0 = \sqrt{\varepsilon_0/\mu_0}\,n_0\cos\theta_0$, $f_s = \sqrt{\varepsilon_0/\mu_0}\,n_s\cos\theta_s$, whereas ε_0 , μ_0 are the permittivity and the permeability of the vacuum respectively. Finally, the transmittance is given by:

$$T = \frac{f_s}{f_0}\left|t^2\right| \tag{13}$$