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Transverse acousto-electric effect in superconductors

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ABSTRACT

We formulate a theory based on the time-dependent Ginzburg–Landau (TDGL) theory and Newtonian vortex dynamics to study the transverse acousto-electric response of a type-II superconductor with Abrikosov vortex lattice. When exposed to a transverse acoustic wave, Cooper pairs emerge from the moving atomic lattice and moving electrons. As in the Tolman–Stewart effect in a normal metal, an electromagnetic field is radiated from the superconductor. We adapt the equilibrium-based TDGL theory to this non-equilibrium system by using a floating condensation kernel. Due to the interaction between normal and superconducting components, the radiated electric field as a function of magnetic field attains a maximum value occurring below the upper critical magnetic field. This local increase in electric field has weak temperature dependence and is suppressed by the presence of impurities in the superconductor.

1. Introduction

Superconductivity appears at low temperatures when materials are rigid and fragile so that for the majority of experiments made in cryostat it is not necessary to consider any motion of the crystal. The standard time-dependent Ginzburg–Landau theory (TDGL), which contains the assumption of local equilibrium, and is formulated in the laboratory frame, is a powerful tool to study phenomena in the vicinity of the superconducting-normal phase transition. To study gyroscopes [1], gravitational wave antennae [2] and the interaction of the superconducting condensate with strong sound waves [3–13], where the atomic lattice is in motion, an extension to TDGL is needed to accommodate the dynamical system.

In the presence of a transverse acoustic wave, the condensate does not experience friction with the crystal and its imperfections as Cooper pairs do not scatter on the underlying crystal or its imperfections. The moving lattice, however, acts on the condensate by three nondissipative mechanisms: (i) Induction: The motion of ions creates an electric current which affects the electrons by magnetic induction. (ii) Entrainment: A Cooper pair of zero momentum in a moving crystal has nonzero velocity with respect to the crystal. This effect is particularly strong in dirty superconductors, where the mass of the Cooper pair is strongly renormalized, $m^* \gg 2m_e$. In the reference frame locally moving with the lattice, entrainment together with the fictitious force compose the inertial force causing the Tolman–Stewart effect. (iii) Deformation potentials: Deformations of the crystal lead to local changes of the chemical potential and material parameters which control superconductivity, for example its critical temperature.

Because of the induction the supercurrent tends to oppose the ionic current, but the compensation is not always complete. For example, in a steadily-rotating (but stationary) superconductor, currents near the surface are only partly compensated and the residual current produces a magnetic field known as the London moment [14]. In non-stationary (oscillating) systems the compensation is even less effective. In particular, under the influence of the ultrasound wave, the inertial motion of normal electrons as well as superconducting electrons leads to nonzero bulk currents via the Tolman–Stewart effect. The oscillating current radiates electromagnetic waves [6,7] so that the superconductor exhibits nonzero acousto-electric effect.

Theoretical analysis of the acousto-electric field in superconductors has been performed assuming the fully superconducting state with no normal current [7]; the normal electrons are ‘clamped’ to the lattice [4,5,11,12]. Using this assumption, the TDGL theory of Verkin and Kulik [15] (originally developed for the case of steady rotation when normal currents are absent) can be used to study the acousto-electric field.

Experimental studies on the acousto-electric field of hole-type metal (niobium) and electron-type borocarbide ($Y_{0.95}Tb_{0.05}Ni_2B_2C$) show a 10% increase of the radiated electric field as the

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material transforms into the superconducting state (Fig. 2 in Ref. [7]). This small change shows that, at least in the vicinity of the transition temperature, the normal and the superconducting currents are comparable, which is incompatible with the assumption of stationary ('clamped') normal electrons. Our aim is to develop a theory which accounts for coexisting normal and superconducting currents.

Here, we study theoretically the effect of a transverse sound wave on a superconducting system near the superconducting-normal phase-transition line, B_{c2}; the transverse wave propagates along the z-axis, and oscillates in the x direction. In the transverse wave the material experiences shear stress only, without compression. According to the experimental studies by Fil et al. [7], the changes of potential and material parameters caused by shear deformations can be disregarded. To accommodate this non-equilibrium system, we use a modified version of TDGL theory based on a microscopic derivation [16].

In Section 2 we formulate the set of TDGL equations for the dynamical system with oscillating atomic lattice driven by an external transverse acoustic wave. Motion of normal electrons is allowed and this current treated in a self-consistent way, instead of assuming the normal electrons move together 'clamped' to the ions as in [4,7]. As a result, the normal current is driven by inertial forces as in the normal-state Tolman–Stewart effect. Details of the derivation based on the Boltzmann equation are given in Appendix A. In Section 3 using Newtonian dynamics, we analyze the acousto-electric effect in the mixed state. Vortex dynamics in a steady state is deduced from the force balance on vortices. Magnus, pinning and transverse forces are considered, along with friction forces from the atomic lattice and from normal electrons [17,18]. The effective forces acting on the superconducting electrons are identified from the extended TDGL equation in Section 2. Next we discuss the skin effect and the matching of the internal field and the radiated electromagnetic wave at the surface. Section 3 concludes with the resulting complete set of equations. Section 4 contains numerical computations of the radiated electric field, using material parameters provided in [7].

2. TDGL theory

To study the electrons in an oscillating atomic lattice, it is advantageous to choose the moving background as the reference frame as Cooper pairs emerge from the moving electrons. Previous study [16] shows that to apply standard TDGL theory to a dynamical system, it is optimal to choose a condensation kernel floating with the background. Here we omit the details and write down the set of equations known as the floating-kernel time-dependent Ginzburg–Landau (FK-TDGL) equations.

Relaxation of the Ginzburg-Landau (GL) order parameter ψ in the dynamical system is described by the FK-TDGL equation

$$\frac{1}{2m^*}(-i\hbar\nabla - e^*\mathbf{A} - \mathbf{P})^2\psi - \alpha\psi + \beta|\psi|^2\psi = \Gamma\left(\frac{\partial}{\partial t} - i\frac{2}{\hbar}\mu\right)\psi \tag{1}$$

with the molecular field

$$\mathbf{P} = \chi^* m^* \dot{\mathbf{u}} + m^* \mathbf{v}_n. \tag{2}$$

The first term of the molecular field is due to the entrainment effect caused by motion of the ionic lattice with velocity $\dot{\mathbf{u}}$; the mass of a Cooper pair is $m^* = m_0/(1+\chi^*)$, where $m_0 = 2m_e$ is twice the electron mass m_e. The corresponding superconducting current is

$$\mathbf{j}_s = \frac{e^*}{m^*}\mathrm{Re}\left[\bar{\psi}(-i\hbar\nabla - e^*\mathbf{A} - \mathbf{P})\psi\right]. \tag{3}$$

The velocity $\mathbf{v}_s$ of the condensate can be defined using $\mathbf{j}_s = e^*|\psi|^2\mathbf{v}_s = en_s\mathbf{v}_s$. Because of the presence of the second term $m^*\mathbf{v}_n$ in $\mathbf{P}$, the operator $(1/m^*)(-i\hbar\nabla - e^*\mathbf{A} - \mathbf{P})$ gives the velocity of Cooper pairs with respect to normal electrons $\mathbf{v}_n$. The current generated by the moving ions is

$$\mathbf{j}_l = -en\dot{\mathbf{u}}, \tag{4}$$

where $\mathbf{u}$ is the ion displacement caused by the transverse sound wave.

In our treatment we relax the requirement of Sonin [4] that the electrons move with the same velocity $\dot{\mathbf{u}}$ as ions. Instead we assume that as with the Tolman–Stewart effect in normal conducting metals [19], the normal electrons lag behind ions and move with velocity $\mathbf{v}_n$. The normal current $\mathbf{j}_n = en\mathbf{v}_n$ can be obtained from the Boltzmann equations, shown in Appendix A; the electric current is

$$\mathbf{j}_n + \mathbf{j}_l = \frac{\sigma_n}{e}\left(\mathbf{F}' - \frac{\tau}{1+i\tau\omega}\frac{e}{m}\mathbf{B}\times\mathbf{F}'\right) - \nu\dot{\mathbf{u}}. \tag{5}$$

The effective driving force

$$\mathbf{F}' = -e\frac{\partial\mathbf{A}}{\partial t} - \nabla\mu + e\dot{\mathbf{u}}\times\mathbf{B} - m_e\ddot{\mathbf{u}} \tag{6}$$

includes the effective electric field (first and second terms), a part of the Lorentz force $\mathbf{F}_L = e(\mathbf{v}' + \dot{\mathbf{u}})\times\mathbf{B}$, where $\mathbf{v}'$ is electron velocity relative to the lattice, and the inertial force. These terms can be understood in the reference frame moving with the lattice, where the third term enters the electric field via a Lorentz transformation. The relaxation time τ comes from the normal conductivity σ_n, from (A.33). The last term in relation (5) results from the diffusion of the transverse momentum [20]; this is similar to the mechanism causing the shear viscosity. Detail derivation of (5), analogous to Ohm's law, from the Boltzmann equation can be found in Appendix A.

From the continuity equation $\nabla\cdot\mathbf{j} = 0$ we can obtain for the chemical potential

$$\nabla^2\mu = \frac{e}{\sigma_n}\nabla\cdot\mathbf{j}_s + e\nabla\cdot(\dot{\mathbf{u}}\times\mathbf{B}), \tag{7}$$

which is simplified by the transversality condition $\mathbf{q}\cdot\mathbf{u} = 0$ for wave vector $\mathbf{q}$. The total force has zero divergence, so $\nabla\cdot\ddot{\mathbf{u}} = 0$. We consider a system with homogeneous conductivity, $\nabla\sigma_n = 0$.

The vector potential $\mathbf{A}$ can be obtained from the Maxwell equation

$$\nabla^2\mathbf{A} = -\mu_0(\mathbf{j}_s + \mathbf{j}_n + \mathbf{j}_l); \tag{8}$$

we use the Coulomb gauge $\nabla\cdot\mathbf{A} = 0$. To obtain the radiated electromagnetic wave, we must evaluate skin vector potential and match internal and external fields. In Section 3.3, we will show that the skin effect is negligible if the wavelength of radiation is much larger than the skin depth.

We have a three-component system consisting of normal electrons, condensate, and electromagnetic field. Eqs. (1), (3), (5), (7) and (8) form a complete set of equations of motion. We are interested below in Section 3 a case that the transverse sound wave interacts with a superconductor in the mixed state. Here we compare our theory with the TDGL theory of Verkin and Kulik [4,15,20] referred as VK-TDGL.

To make the comparison, we rewrite our equations in terms of the relative velocities with respect to the atomic lattice, that is, the relative velocity of normal electrons as $\mathbf{v}_n' = \mathbf{v}_n - \dot{\mathbf{u}}$ and the relative velocity of the condensate as $\mathbf{v}_s' = \mathbf{v}_s - \dot{\mathbf{u}}$.

In this notation, the molecular field (2) is

$$\mathbf{P} = m_0\dot{\mathbf{u}} + m^*\mathbf{v}_n'. \tag{9}$$

The first term is the fictitious force obtained by Verkin and Kulik [15]. The second term which is absent in [15] is a correction due to non-zero velocity of normal electrons with respect to the ionic lattice. From Ohm's law (5), (A.33) and (A.36), we can see that the