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# Proposed experimental test of the theory of hole superconductivity

J.E. Hirsch*

*Department of Physics, University of California, San Diego, La Jolla, CA 92093-0319, United States*

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ABSTRACT

The theory of hole superconductivity predicts that in the reversible transition between normal and superconducting phases in the presence of a magnetic field there is charge flow in direction perpendicular to the normal-superconductor phase boundary. In contrast, the conventional BCS-London theory of superconductivity predicts no such charge flow. Here we discuss an experiment to test these predictions.



## 1. Introduction

Since the Meissner-Ochsenfeld [1] experimental discovery in 1933 that superconductors expel magnetic fields, and Gorter and Casimir's [2] theoretical analysis shortly thereafter, it has been known that the transformation between normal and superconducting states in the presence of a magnetic field is a reversible phase transformation between equilibrium phases of matter. The physics is described phenomenologically by the London electrodynamic equations [3,4] and microscopically by the BCS theory of superconductivity [5]. However we have argued in recent work that neither London's theory nor BCS explain the *process* by which a normal metal becoming superconducting expels a magnetic field [6,7], nor the process by which a superconductor in a magnetic field transforming to the normal phase loses its screening current without dissipation of Joule heat [8]. We have furthermore argued that to explain these phenomena some physical elements are needed that are not part of the conventional BCS-London theory of superconductivity but are part of the theory of hole superconductvity. [9]

Let us summarize the shortcomings of the conventional theory: in the transition from the normal to the superconducting state, the process of expelling the magnetic field generates a Faraday electric field that applies a force on carriers that is opposite to the direction of the Meissner current that is generated to expel the magnetic field. In other words, the process has to overcome the counter-emf that gets generated as magnetic field lines move out. The conventional theory argues that the free energy of the system is lowered in the process, but does not explain the physical origin of the *force* that propels the process. Another way to say it, the carriers of the Meissner current have to acquire a mechanical momentum opposite to the direction of the Faraday force, and the conventional theory does not explain how this occurs. In addition, the body as a whole has to acquire an opposite momentum, also opposite to the direction of the Faraday force acting on it, so that the total momentum is conserved, and there is no explanation for how this happens in the conventional theory [10]. In the reverse process where a superconductor in a magnetic field becomes normal, the conventional theory does not explain how the momentum of the extinguishing supercurrent is transferred to the body as a whole without collisions that would result in dissipation of Joule heat in contradiction with the known reversibility of the process [8].

The theory of hole superconductivity overcomes these difficulties because it predicts that there is a flow and counterflow of charge in direction perpendicular to the phase boundary as the phase boundary moves with speed $\dot{x}_0$, as described in [6] and shown schematically in Fig. 1. A carrier that is becoming superconducting (electron) thrusts forward at a high speed $v_x$ ($v_x \gg \dot{x}_0$) (labeled 'flow' in the figure) and acquires through the magnetic Lorentz force a velocity in the $+\hat{y}$ direction (dotted arrow), thus generating the supercurrent $J_y$ in the $-\hat{y}$ direction. A counterflow of normal negative charge occurs in the $-\hat{x}$ direction (not shown in the figure), which is equivalent (as shown in the figure) to a *hole* current moving in the $+\hat{x}$ direction (labeled 'counterflow' in the figure) at speed $\dot{x}_0$. This hole counterflow transfers momentum to the lattice ($P_{latt}$) that exactly cancels the momentum acquired by the supercurrent. The counterflow occurs within a boundary layer of thickness $\lambda_L$ from the phase boundary ($\lambda_L$ = London penetration depth). For the reverse process where the phase boundary moves in the $-\hat{x}$ direction expanding the normal phase the same processes occur with the directions of the arrows reversed. For more details see refs. [6–8].

* Tel.: +1 858 534 3931.
*E-mail address:* jhirsch@ucsd.edu

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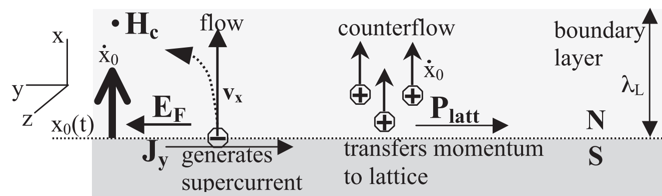


**Fig. 1.** Schematics of charge flow and counterflow in direction perpendicular to the normal (N) – superconductor (S) phase boundary, as the phase boundary located at $x = x_0(t)$ (dotted horizontal line) moves upward towards the normal region ($+\hat{x}$ direction) with speed $\dot{x}_0$. The magnetic field $H_c$ points out of the paper. $E_F = H_c(\dot{x}_0/c)$ is the Faraday electric field generated by the changing magnetic flux due to the moving phase boundary. For a detailed explanation see text.

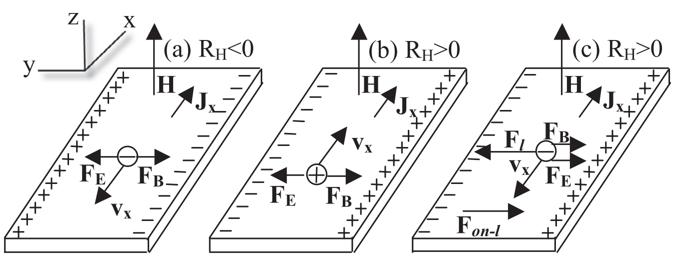


**Fig. 2.** Hall effect geometry representative of the counterflow process shown in Fig. (1). In (a) the normal state carriers are assumed to be electrons, in (b) and (c) holes. $F_E$ and $F_B$ are electric and magnetic forces on the carrier, $F_l$ is the force exerted by the lattice on the carrier and $F_{on-l}$ is the force exerted by the carrier on the lattice. For a detailed explanation see text.

In the following section we analyze in detail why it is crucial that the normal state carriers are holes in order to explain the physics. In Section 3 we discuss how this physics can be detected experimentally. We conclude in Section 4 with a discussion.

## 2. Why hole carriers are necessary for superconductivity

We consider the flow of current in crossed electric and magnetic fields as shown in Fig. 2, since this is the situation that takes place when a superconductor expels the magnetic field. This is the standard geometry of the Hall effect. Fig. 2(a) shows a situation where the Hall coefficient is negative. As shown by Ashcroft and Mermin [11], when all occupied k-space orbits are closed the Hall coefficient takes the simple form

$$R_H = \frac{E_y}{J_x H} = \frac{1}{nec} \tag{1}$$

with

$$n = \int_{occ} \frac{d^3k}{4\pi^3} \tag{2a}$$

$$J_x = \int_{occ} \frac{d^3k}{4\pi^3} v_{k,x} \tag{2b}$$

$$\vec{v}_k = \frac{1}{\hbar}\frac{\partial \epsilon_k}{\partial \vec{k}} \tag{2c}$$

The semiclassical equation of motion gives for the force in the $\hat{y}$ direction

$$m_e \frac{dv_{k,y}}{dt} = eE_y - \frac{e}{c} v_{k,x} H + F_{l,y} \tag{3}$$

where $m_e$ is the bare electron mass and $\vec{F}_l$ is the force exerted by the lattice on the electrons. In the presence of scattering we replace $dv_{k,\,y}/dt$ by $v_{k,\,y}/\tau$ on the left side, with $\tau$ the collision time, and integrating over the occupied $k$-states and using Eq. (2) yields

$$\frac{m_e}{e}\frac{J_y}{\tau} = enE_y - \frac{H}{c} J_x + \int_{occ} \frac{d^3k}{4\pi^3} F_{l,y} \tag{4}$$

Setting $J_y = 0$ as appropriate for the geometry in Fig. 2 and using Eq. (1) then yields

$$\int_{occ} \frac{d^3k}{4\pi^3} F_{l,y} = 0 \tag{5}$$

Therefore, the total force exerted by the lattice on the conduction electrons in the $\hat{y}$ direction is zero, and consequently the total force exerted by the conduction electrons on the lattice in the $\hat{y}$ direction is zero. This indicates that normal state electron carriers moving in crossed electric and magnetic fields will not transfer momentum to the lattice in the absence of scattering. However, both in the process of a superconductor becoming normal in the presence of a magnetic field [8], or in the reverse process of the superconductor expelling a magnetic field [10], it is necessary that the electrons transfer momentum to the lattice in a reversible way, without scattering processes. Therefore we conclude that superconductivity as we know it cannot occur if the normal state carriers are electrons.

The situation is different if the normal state carriers are holes and the Hall coefficient is positive, shown in Fig. 2(b), (c). As shown by Ashcroft and Mermin [11] if all the *unoccupied* orbits are close the Hall coefficient is given by

$$R_H = \frac{E_y}{J_x H} = \frac{1}{n_h |e| c} \tag{6}$$

where

$$n_h = \int_{unocc} \frac{d^3k}{4\pi^3} = \frac{2}{v} - n \tag{7}$$

is the number of hole carriers, i.e. of empty states in the band, and $v$ is the volume of the unit cell. We rewrite Eq. (4) as

$$\frac{m_e}{e}\frac{J_y}{\tau} = e\left(\frac{2}{v} - n_h\right)E_y - \frac{H}{c} J_x + \int_{occ} \frac{d^3k}{4\pi^3} F_{l,y}, \tag{8}$$

and setting $J_y = 0$ and using Eq. (6) yields

$$F_l = \int_{occ} \frac{d^3k}{4\pi^3} F_{l,y} = |e| E_y \frac{2}{v} \tag{9}$$

for the total force exerted by the lattice on the electrons in the $\hat{y}$ direction. Correspondingly the force exerted by the electrons on the lattice is

$$F_{on-l} = -F_l = \int_{occ} \frac{d^3k}{4\pi^3} F_{l,y} = -|e| E_y \frac{2}{v} \tag{10}$$

in the $-\hat{y}$ direction, i.e. to the right. This is the direction required to transfer momentum to the lattice as shown in Fig. 1.

In addition, there is the force exerted by the electric field on the ions, which points to the left and substracts from Eq. (10). Since the system is charge neutral we can assume that the concentration of ions is $(2/v - n_h)$ equal to the concentration of electrons, so that the net force exerted on the ions is

$$F^{net}_{on-l} = -|e| E_y \frac{2}{v} + |e| E_y \left(\frac{2}{v} - n_h\right) = -|e| n_h E_y. \tag{11}$$

This result can also be deduced simply by looking at Fig. 2(c) and substracting from $F_l$ the force exerted on the electron due to the electric field, which gives for each carrier a force $|e|E_y = F_l/2$ pointing to the left, hence the same force pointing to the right for the net force exerted on the lattice per carrier. For the case of Fig. 2(a) there is no force exerted by the electrons on the lattice but there is a force on the lattice due to the electric field $E_y$, yielding the same result as Eq. (11) with $n$ replacing $n_h$. Note that these results agree with what is predicted by Ampere's force law

$$\vec{F} = \frac{I}{c} \vec{L} \times \vec{H} \tag{12}$$

for the force on a Hall bar of length $L$ and total current $I$, which is of course independent of whether the carriers are electrons or holes.