Physica C: Superconductivity and its applications 525–526 (2016) 52–55

Contents lists available at ScienceDirect

Physica C: Superconductivity and its applications

journal homepage: www.elsevier.com/locate/physc

The critical current of point symmetric Josephson tunnel junctions

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ARTICLE INFO

Article history:
Received 3 April 2016
Revised 19 April 2016
Accepted 20 April 2016
Available online 23 April 2016

Keywords:
Josephson effect
Magnetic diffraction pattern
Point symmetric junctions

ABSTRACT

The physics of Josephson tunnel junctions drastically depends on their geometrical configurations. The shape of the junction determines the specific form of the magnetic-field dependence of its Josephson current. Here we address the magnetic diffraction patterns of specially shaped planar Josephson tunnel junctions in the presence of an in-plane magnetic field of arbitrary orientations. We focus on a wide ensemble of junctions whose shape is invariant under point reflection. We analyze the implications of this type of isometry and derive the threshold curves of junctions whose shape is the union or the relative complement of two point symmetric plane figures.

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1. Introduction

Any Josephson device is characterized by a maximum zero-voltage d.c. current, I_c, called critical current, above which it switches to a finite voltage. How the critical current modulates with an external magnetic field is an important issue for all the earlier [1] and novel [2] applications of the Josephson effect. It has long been addressed that the magnetic diffraction pattern (MDP) of planar Josephson tunnel junctions (JTJs) drastically depends on both the specific shape of the tunneling area and the direction of the in-plane applied field. Most of the milestone works which allowed significant advances in the understanding of the geometrical properties of the MDP [3–5] just considered a number of interesting shapes with the magnetic field applied in a preferential direction. However, the knowledge of the MDP for arbitrary field direction allows to evaluate the consequences of an unavoidable field misalignment in the experimental setups. Moreover, the measurements of $I_c(H)$ provides the first quality test of any Josephson device.

In this Letter we highlight the MDP properties of a wide class of JTJs characterized by a point symmetric shape in the general case of an arbitrarily oriented in-plane magnetic field. Furthermore, in force of the additive property of the surface integrals, we show that the threshold curves of JTJs with complex shapes can be expressed as a linear combination of the MDP of junctions with simpler point symmetric shapes.

1.1. Small JTJs

In Josephson's original description the quantum mechanical phase difference, ϕ, across the barrier of a generic two-dimensional planar Josephson tunnel junction is related to the magnetic field, $\mathbf{H}$, inside the barrier through [6]:

$$\nabla \phi = \kappa \mathbf{H} \times \mathbf{u}_z, \tag{1}$$

in which $\mathbf{u}_z$ is a unit vector orthogonal to the junction plane and $\kappa \equiv 2\pi \mu_0 d_m / \Phi_0$, where Φ_0 is the magnetic flux quantum, μ_0 the vacuum permeability, and d_m the junction *magnetic* penetration depth [7,8]. The external field $\mathbf{H}$, in general, is given by the sum of an externally applied field and the self-field generated by the current flowing in the junction. If the junction dimensions are smaller than the Josephson penetration length, the self-magnetic field is negligible, as has been first shown by Owen and Scalapino [9] for a rectangular JTJ . Henceforth, for (electrically) small JTJs the phase spatial dependence is obtained by integrating Eq. (1); in Cartesian coordinates, for an in-plane magnetic field applied at an arbitrary angle θ with the Y-axis, $\mathbf{H} = (H\sin\theta, H\cos\theta)$, it is:

$$\phi(x, y, H, \theta, \phi_0) = \kappa H (x\cos\theta - y\sin\theta) + \phi_0, \tag{2}$$

where ϕ_0 is an integration constant. The tunnel current flows in the Z-direction and the local density of the Josephson current is [6]:

$$J_J(x, y, H, \theta, \phi_0) = J_c \sin\phi(x, y, H, \theta, \phi_0), \tag{3}$$

where the maximum Josephson current density, J_c, is assumed to be uniform over the junction area. The Josephson current, I_J, through the barrier is obtained integrating Eq. (3) over the junction surface, S:

$$I_J(H, \theta, \phi_0) = \int_S J_J \, dS = J_c \int_S \sin\phi \, dS. \tag{4}$$

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http://dx.doi.org/10.1016/j.physc.2016.04.009
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The junction critical current, I_c, is defined as the largest possible Josephson current, namely,:

$$I_c(H,\theta) = \max_{\phi_0} I_J(H,\theta,\phi_0), \tag{5}$$

The integral in Eq. (4) applied to the surface of an axis-parallel rectangle yields:

$$\int_{y_1}^{y_2} dy \int_{x_1}^{x_2} dx \sin(k_x x - k_y y + \phi_o) = \frac{4}{k_x k_y} \sin\frac{k_x(x_2 - x_1)}{2} \times \sin\frac{k_y(y_2 - y_1)}{2} \sin\left[\frac{k_x(x_2 + x_1)}{2} - \frac{k_y(y_2 + y_1)}{2} + \phi_0\right], \tag{6}$$

where (x_1, y_1) and (x_2, y_2) are the coordinates of, respectively, the lower left and upper right rectangle corners. Identifying k_x and k_y with, respectively, $\kappa H\cos\theta$ and $\kappa H\sin\theta$, the critical current is achieved when $2\phi_0 = \pm\pi + k_y(y_2 + y_1) - k_x(x_2 + x_1)$ and we end out with the well-known double Fraunhofer diffraction pattern of a small rectangular JTJ in an arbitrarily oriented magnetic field [1]:

$$I_c^R(H,\theta) = J_c A_R \left| \frac{\sin[(\kappa HW\sin\theta)/2]}{(\kappa HW\sin\theta)/2} \frac{\sin[(\kappa HL\cos\theta)/2]}{(\kappa HL\cos\theta)/2} \right|, \tag{7}$$

where $W = x_2 - x_1$ and $L = y_2 - y_1$ are the rectangle edges and $A_R = WL$ its area. It can be be demonstrated that, if the rectangle is rotated by an arbitrary angle γ with respect to the Y-axis, as intuitively expected, Eq. (7) transforms to:

$$I_c^R(H,\theta) = J_c A_R \left| \frac{\sin\{[\kappa HW\sin(\theta-\gamma)]/2\}}{[\kappa HW\sin(\theta-\gamma)]/2} \times \frac{\sin\{[\kappa HL\cos(\theta-\gamma)]/2\}}{[\kappa HL\cos(\theta-\gamma)]/2} \right|.$$

The quantity within the absolute-value bars in Eq. (7) can be thought of as characteristic area-independent function, $\mathcal{F}_R(H,\theta)$, of all the axis-parallel rectangles with aspect ratio W/L:

$$\mathcal{F}_R(H,\theta) \equiv \frac{\sin[(\kappa HW\sin\theta)/2]}{(\kappa HW\sin\theta)/2} \frac{\sin[(\kappa HL\cos\theta)/2]}{(\kappa HL\cos\theta)/2}. \tag{8}$$

2. Complementary junctions - Concentric rectangles

To state the problem, let us consider a small planar JTJ whose tunneling area is obtained as the difference between two concentric and parallel rectangles of arbitrary aspect ratios such that the smaller rectangle, r, lies wholly inside the outer rectangle, R ($r \subset R$). This geometry is depicted in Fig. 1 where the origin of the Cartesian axes is in the center of the junction; the sides lengths of the outer (inner) rectangle are $W(w)$ and $L(l)$. Such a structure can be realized by etching away or anodizing the inner part of the junction base electrode. The surface of this complementary junction can be decomposed in the four rectangles shown in gray; we can, therefore, calculate its MDP, $I_c^{R-r}(H,\theta)$, making use of Eq. (6) for a single rectangle and exploiting the additive property of integrals. After some algebraic manipulation, one founds that for any value of the field angle θ:

$$I_c^{R-r}(H,\theta) = J_c|A_R\mathcal{F}_R(H,\theta) - A_r\mathcal{F}_r(H,\theta)|, \tag{9}$$

where $A_R = WL$, $A_r = wl$ and the characteristic functions $\mathcal{F}_r$ and $\mathcal{F}_R$ are defined through the MDPs of the fictitious inner and outer rectangular junctions, respectively, $I_c^r(H,\theta) \equiv J_c A_r|\mathcal{F}_r(H,\theta)|$ and $I_c^R(H,\theta) \equiv J_c A_R|\mathcal{F}_R(H,\theta)|$. Interestingly, Eq. (9) still holds if the rectangles are not parallel. Moreover, a similar expression also applies when one or both the rectangles are replaced by arbitrarily oriented concentric rhombuses or ellipses.

We remind that: (i) for a small diamond-like JTJ of diagonals P and Q parallel to Cartesian axes the characteristic function is [10]:

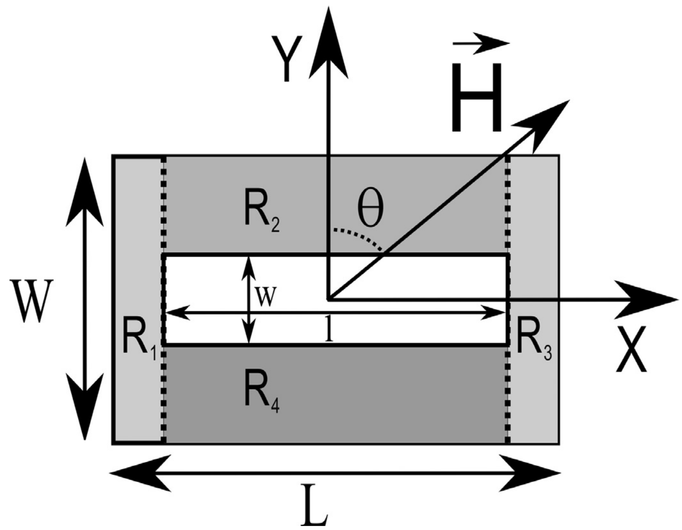

Fig. 1. Schematic of a complementary planar JTJ resulting by the difference between two concentric and axis-parallel rectangles of arbitrary aspect ratios. The outer rectangle has sides of lengths L and W, while the inner one has sides of lengths l and w. The junction area, $WL - wl$, is given by the sum of the areas of the four gray rectangles. The in-plane magnetic field, **H**, is applied at a generic angle, θ, with the Y-axis.

$$\mathcal{F}_D(H,\theta) = 2\frac{\cos[(\kappa HP\sin\theta)/2] - \cos[(\kappa HQ\cos\theta)/2]}{(\kappa HP\sin\theta/2)^2 - (\kappa HQ\cos\theta/2)^2} \tag{10}$$

and (ii) for a planar JTJ delimited by an axis-aligned ellipse of principal semi-axes a and b it is [11]:

$$\mathcal{F}_E(H,\theta) = 2\frac{J_1[\kappa Hp_E(\theta)/2]}{\kappa Hp_E(\theta)/2}, \tag{11}$$

where J_1 the 1st order Bessel function of the first kind and $p_E(\theta) \equiv 2\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$ the length of the projection of the ellipse in the direction normal to the externally applied magnetic field. Eq. (11), first reported by Peterson et al. [11] in 1990, generalizes the so called *Airy pattern* of a circular junction [1] of radius $r = a = b$.

Indeed, we found that the MDP of a complementary JTJ resulting from the difference (complement), $s' = S - s$, of two concentric plane figures with two lines of symmetry (including unconventional shapes like, for example, crosses, bow-ties, s-shapes and figure-eights), can be expressed in terms of their areas, A_S and A_s, and characteristic functions, $\mathcal{F}_S$ and $\mathcal{F}_s$, that is:

$$I_c^{s'}(H,\theta) = J_c|A_S\mathcal{F}_S(H,\theta) - A_s\mathcal{F}_s(H,\theta)|. \tag{12}$$

A similar expression was proved for the sum (union), S, of two disjoint figures, s and s', namely:

$$I_c^S(H,\theta) = J_c|A_s\mathcal{F}_s(H,\theta) + A_{s'}\mathcal{F}_{s'}(H,\theta)|. \tag{13}$$

In the following, we will demonstrate that the broadest geometrical requirement for the validity of Eqs. (12) and (13) is the point-symmetry of the plane figures.

3. Point symmetric Josephson tunnel junctions

Let us consider a small JTJ whose shape has a second order *point* or *central-inversion* symmetry, that is to say, is invariant upon a 180° rotation around one point called center of symmetry, namely upon reflections in two perpendicular lines. If we pick any Cartesian system with origin in the center of symmetry, then the figure contour in the first (second) quadrant is reproduced specularly in the third (fourth) quadrant. One example of point-symmetric figure is illustrated by the gray shape in Fig. 2. Any line

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