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Effect of non-uniform surface resistance on the quality factor of superconducting niobium cavity

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ABSTRACT

The formula $R_s = G/Q_0$ is commonly used in the calculation of the surface resistance of radio frequency niobium superconducting cavities. The applying of such equation is under the assumption that surface resistance is consistent over the cavity. However, the distribution of the magnetic field varies over the cavity. The magnetic field in the equator is much higher than that in the iris. According to Thermal Feedback Theory, it leads non-uniform distribution of the density of heat flux, which results in a different temperature distribution along the cavity inter surface. The BCS surface resistance, which depends largely on the temperature, is different in each local inner surface. In this paper, the effect of surface non-uniform resistance on the quality factor has been studied, through the calculation of Q_0 in the original definition of it. The results show that it is necessary to consider the non-uniform distribution of magnetic field when the accelerating field is above 20 MV/m for TESLA cavities. Also, the effect of inhomogeneity of residual resistance on the quality factor is discussed. Its distribution barely affects the quality factor.

1. Introduction

The ultra-high quality factor Q_0 is the most excellent feature of superconducting radio frequency cavities, which is almost ten thousand times than normal conducting structures. The main challenge in the performance improvement of SRF cavities is achieving a high Q_0 in high accelerating field gradient. Recently, nitrogen-doping method has been approved as an effective way to increase Q_0. However, those nitrogen-doped cavities quenched in the medium field even its Q_0 conserved $5 \sim 6 \times 10^{10}$ [1]. Many experts tried to explain the reasons of good performance of nitrogen-doping cavities. In those theoretical researches of nitrogen-doping, an important method is calculating the surface resistance by using the equation $R_s = G/Q_0$ [2,3]. Except that, in the research about low temperature baking [4], medium field Q slope [5,6] and grain boundaries [7] in SRF cavities, the equation $R_s = G/Q_0$ has been used widely to obtain the surface resistance of SRF cavities. When using this equation, it is necessary to assume that the surface resistance does not vary over the cavity surface [8]. For surface resistance of SRF cavities, it consists of BCS resistance and residual resistance. BCS resistance is introduced by BCS theory, which is strongly temperature depended. Residual resistance is resulted of foreign material inclusions, grain boundaries and so on, These defects trap vortices [14] which is the main reason of residual resistance.

The energy dissipated in the surface is associated with the magnetic field, such is $P_{diss} \sim |H|^2$. The inhomogeneity of magnetic field will lead to inhomogeneity of temperature over the interior surface of cavity. Consequently, the BCS resistance varies around the surface. Fig. 1 shows that surface temperature changes along with the changes of magnetic field by using Thermal Feedback Model [5,6]. The Thermal Feedback Model will be explained in detail in this article. Fig. 2 shows that BCS resistance varies as the magnetic field changes. The BCS resistance is calculated by Srimp Code [9] via knowing the surface temperature from Fig. 1. Fig. 2 shows that the BCS resistance grows bigger and bigger as the magnetic field increases.

From Fig. 2, the resistance in 50 mT is slightly more than that in 0 mT. This difference is small enough to be ignored. However, the resistance in 150 mT is about four times than that in 0 mT. This is a huge number that cannot be neglected. Thus, if the magnetic field in the cavity is small enough to ignore the variation of BCS resistance, the equation $R_s = G/Q_0$ is a good way to get the surface resistance of SRF cavities. But in some circumstances such as low temperature baking or medium field Q slope and so on, its involving magnetic field is higher than 80 mT. The variation of BCS resistance resulted from magnetic field inhomogeneity need be studied.

The relation between quality factor Q_0 and magnetic field B is complex because it has three distinct regions: (1) low field Q-slope (LFQS) part: the Q_0 stays flat or increases as B increases; (2)

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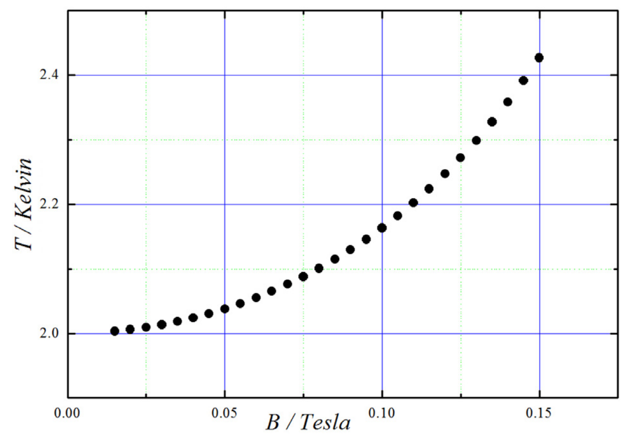

Fig. 1. The surface temperature as a function of the magnetic field. The surface temperature is calculated by the Thermal Feedback Model.

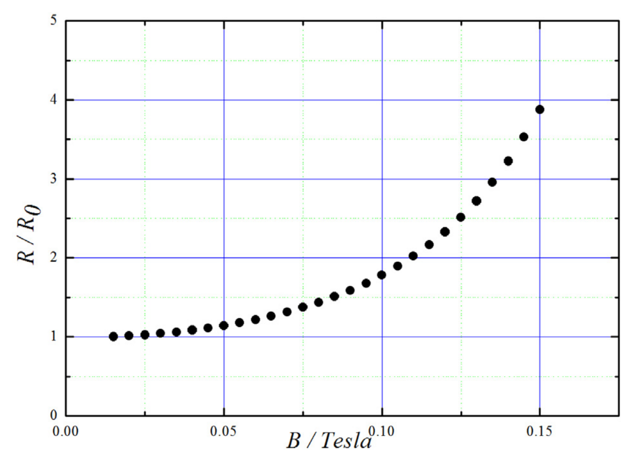

Fig. 2. The R_0 represents the surface resistance in 0 mT. The ratio between surface resistance in different magnetic field and R_0 is obtained by Srimp Code using the temperature from Fig. 1.

medium field Q-slope (MFQS) part: the Q_0 gradually decreases by a factor of 2 or so; (3) high field Q-slope (HFQS) part: Q_0 sharply decreases and the cavity turns out to quench. Various models were raised to explain those slopes. For LFQS, it is poorly understood since it is un-relevant in most srf application. But it bring many experts' interests [3] because of N-doping. For MFQS, Thermal Feedback Model is a good theory to explain it. For HFQS, surface defects, grain boundaries, the formation of hydrides and many other factors contribute to this slope. But for some cavity with good post-processing, the HFQS is not obvious [10]. In this paper, Thermal Feedback Model is used to estimate non-uniform magnetic field's influences on the quality factor Q_0. For N-doped cavity, there are no agreements on its characteristics. So only cavities without N-doping were take consideration in the calculating of quality factor.

2. The calculation of quality factor

As we know, the quality factor Q_0 is defined as

$$Q_0 = \frac{\omega_0 U}{P_c} \tag{1}$$

where U is the stored energy and P_c is the power dissipated in the cavity walls. ω_0 is resonat frequency. The total energy stored by a cavity can be written as:

$$U = \frac{1}{2}\mu_0 \int_V |H|^2 dV \tag{2}$$

where the integral is taken over the volume of the cavity. Besides, the dissipated power can be given as:

$$P_c = \frac{1}{2}\int_S R_s |H|^2 dS \tag{3}$$

where the integral is taken over the interior surface of the cavity. Thus, the quality factor can be shown as:

$$Q_0 = \frac{\omega_0 \frac{1}{2}\mu_0 \int_V |H|^2 dV}{\frac{1}{2}\int_S R_s |H|^2 dS} \tag{4}$$

If the relevance between surface resistance R_s and magnetic field H is ignored. Therefore, we could obtain:

$$Q_0 = \frac{G}{R_s} \tag{5}$$

where G is known as geometry factor, which is a constant for a particular cavity in a particular mode [8].

The quality factor can be calculated in another method by calculating the integration in formula (4). To computing the integration, discretizing it is an effective method to obtain its value. Take the TESLA cavities as an example, and the quality factor can be written as:

$$Q_0 = \frac{\omega_0 U}{\frac{1}{2}\int_S R_s |H|^2 dS} \cong \frac{\omega_0 U}{\frac{1}{2}\sum_i R_s(i) \cdot |H_i|^2 \cdot S_i} \tag{6}$$

In order to calculate Q_0, it is essential to get the magnetic field H_i and its corresponding surface area S_i in every discretized part. The surface resistance $R_s(i)$ is decided by its corresponding magnetic field in a given cavity. For simplicity, only cavity with uniform residual resistance is considered.

According to the design of TESLA cavity [11], it consists of three main parts, which are ellipse in iris, straight line for transition, circular arc in equator. Such are shown in Fig. 3 in two-dimension plane. The parameters of TESLA cavity are also shown in Fig. 3.

CST software has been adopted to simulate the distribution of magnetic field. The construction of model for TESLA cavity in CST is followed by parameters in Fig. 3. Through the Eigenmode Analysis in CST, the distribution of magnetic field is given in Fig. 4. In simulations, the energy storied in the cavity is 1 J, the average accelerating field $E_{acc} = 10.07$ MV/m and the peak magnetic field is 33770 A/m.

From Fig. 4, which is generated by CST software, the magnetic field in equator is much higher than that in iris. Because mesh cells used is Hexahedral and it is not axially-symmetric. So it leads to some unusual spots or streaks in the distribution of magnetic field. In Fig. 6, the errorbars were used to indicate those unusual parts' influences.

The distribution of the magnetic field in TESLA is axial symmetric because of its axisymmetric shape. In order to discretizing the integration in formula (4), we could divide the cavity space into many axisymmetric parts along the axis. The two-dimension picture Fig. 5 shows the method of discretizing.

Because of axisymmetric features of Tesla cavity, this brings some conveniences to get its magnetic field distribution. If every part (x_{i-1}, x_i) is so thin that the value of magnetic field H_i can be regarded as the same. The value of magnetic field H_i of every part could be obtained by CST software.

Fig. 6 represents the value of magnetic field in every thin part. The errorbar in Fig. 6 shows the standard deviation of magnetic in every part. Compared with the 10,000 A/m in iris, the magnetic field in equator is about 27,000 A/m. This indicates the non-uniform distribution of magnetic field around the inner surface of