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# The fluctuation Hall conductivity and the Hall angle in type-II superconductor under magnetic field

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ABSTRACT

The fluctuation Hall conductivity and the Hall angle, describing the Hall effect, are calculated for arbitrary value of the imaginary part of the relaxation time in the frame of the time-dependent Ginzburg–Landau theory in type II-superconductor with thermal noise describing strong thermal fluctuations. The self-consistent Gaussian approximation is used to treat the nonlinear interaction term in dynamics. We obtain analytical expressions for the fluctuation Hall conductivity and the Hall angle summing all Landau levels without need to cutoff higher Landau levels to treat arbitrary magnetic field. The results are compared with experimental data on high-$T_c$ superconductor.



## 1. Introduction

The Hall effect has received considerable experimental and theoretical attention [1–11] after the discovery of high-Tc superconductors (HTSCs). The interesting features in the Hall effect of HTSCs is the sign reversal of Hall coefficient $R_H$ in magnetic fields at the temperature just below the superconducting transition temperature. This feature is detected in many HTSCs [3,5,7] and even in some conventional superconductors [12,13]. Moreover, a double-sign reversal in mixed states, which is a subsequent return of the Hall resistivity to the positive value before vanishing, has been observed in highly anisotropic HTSCs such as $Bi_2Sr_2CaCu_2O_x$ crystals [8], or $HgBa_2CaCu_2O_6$ films [9]. The existence of the second sign change was also reported $YBa_2Cu_3$-$O_{7-x}$ (YBCO) films [10,14]. Finally, even a triple-sign reversal was reported in $HgBa_2CaCu_2O_6$ films with columnar defects induced by high-density ion irradiation [11].

A variety of theories were proposed to explain the complex features of the Hall resistivity temperature dependence, but consensus has not been achieved yet. In HTSCs, the Hall anomaly may be due to the pinning force [15], nonuniform carrier density in the vortex core [16,17], or can be calculated in the time dependent Ginzburg–Landau (TDGL) model [1,2,4]. Several theoretical approaches claim to predict the double or triple-sign reversal, based either on entirely intrinsic mechanism of vortex motion and electronic spectrum [18], or on hydrodynamic interaction between vortices and the superconducting and normal-state fluids [19]. The second sign change of the Hall resistivity turned out to be strongly vortex-pinning dependent, since it vanished at high transport current densities, or with the magnetic field B tilted off the twin boundaries by a small angle ($5^o$) [20]. Based on a simple model of pinning potential, Kopnin and Vinokur [21] showed that an increasing pinning strength not only affected the longitudinal flux-flow resistivity, but also decreased the magnitude of the vortex contribution to the Hall voltage (fluctuation term in the TDGL approach). A strong enough pinning can even result in a second sign reversal of the Hall resistivity [21,22]. However, for temperatures near the critical region where the first sign change of the Hall resistivity occurs, the pinning contribution to the Hall conductivity is almost negligible [20]. The TDGL approach is therefore considered to be appropriate, but theory should be modified by including pinning effects at lower temperatures and magnetic fields.

According to the TDGL formalism, the total Hall conductivity is sum of the difference in sign between the normal part and the superconducting fluctuation part. These two parts have opposite signs, if the energy derivative of the density of states averaged over the Fermi surface is positive when the carriers are holes in the normal state [23]. Therefore, the sign reversal can be intrinsic and depends on the details of the structure of the electronic states at the Fermi surface. Although admittedly fluctuation Hall conductivity $\sigma^s_{xy}$ arises a result of an electron-hole asymmetry in the band structure [24], it turns out that this is not the unique source of the appearance of the imaginary part of the relaxation time in the TDGL equation. It was shown that the sign and value of the

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imaginary part of the relaxation time strongly depend also on the topology of Fermi surface [25]. Recently, the Hall coefficient $R_H$ and the Hall angle $\theta_H$ of the cuprate superconductor YBCO was measured in high field in the underdoped regime [6,7], which reveals that the change of sign in the Hall coefficient $R_H$ is attributed to the emergence of electron pockets in the Fermi surface. These electron pocket are not supported by the band structure of YBCO, so they must come from a reconstruction of the Fermi surface. Moreover, value of the imaginary part of the relaxation time is not small in comparison with the real one in serveral HTSCs [4]. However, an additional assumption often made in analytical calculations [1,4] that the most dominant contribution to Hall conductivity $\sigma^s_{xy}$ was the first order term in the imaginary part of the relaxation time.

In this paper the fluctuation Hall conductivity and the Hall angle in two dimensional (2D) and three dimensional (3D) model are calculated by using the time TDGL approach for arbitrary value of the imaginary part of the relaxation time with strong thermal fluctuations. Self-consistent Gaussian approximation used in this paper is consistent to leading order with perturbation theory [26] in which it is shown that this procedure preserved a correct the ultraviolet (UV) renormalization (is renormalization group invariant). Without electric field the issue was comprehensively discussed in a textbook of Kleinert [27]. While the Hartree method is generally simpler, the Gaussian method applied in its consistent form conserves Ward identities (electric current) and its effective energy is positive definite. In addition it has the correct large number of components limit, unlike Hartree method. A main contribution of our paper are explicitly analytical formulas of the fluctuation Hall conductivity and the Hall angle incorporating all Landau levels and including arbitrary value of the imaginary part of the relaxation time.

The paper is organized as follows. The model is defined in Section 2. The Hall conductivity and the Hall angle in 2D is described in Section 3, while extension to 3D model is Section 4. The comparison with experiment is described in Section 5. Finally, we conclude in Section 6.

## 2. The time dependent Ginzburg–Landau in 2D model

We apply this model to describe experiments not just in BiSCCO (highly anisotropic material) but also in underdoped YBCO. For more isotropic optimally doped or fully doped YBCO, an anisotropic 3D GL model (neglecting the layered structure) would be more appropriate. The imaginary part of the relaxation time $\Gamma_0^{-1}$ in the TDGL equation must be introduced to break the particle-hole symmetry and allow for a nonvanishing Hall current [4]. The gauge-invariant relaxational TDGL equation governing the critical dynamics of the superconducting order parameter takes form:

$$\Gamma_0^{-1}(1+i\lambda)\left(\frac{\partial}{\partial\tau}-i\frac{e^*}{\hbar}\Phi\right)\Psi-\frac{\hbar^2}{2m^*}\left|\nabla+i\frac{2\pi}{\Phi_0}\mathbf{A}\right|^2\Psi$$
$$+a\Psi+b|\Psi|^2\Psi=\zeta, \tag{1}$$

Here $m^*$ is effective Cooper pair mass in the ab plane, $a=a_0(t-1)$ is the GL potential with $t=T/T_0$. The "mean field" critical temperature $T_0$ depends on UV cutoff of the "mesoscopic" or "phenomenological" GL description. The vector potential describes constant and homogeneous magnetic field $\mathbf{A}=(-By,0)$ and $\Phi_0=hc/e^*$ is the flux quantum with $e^*=2|e|$.The electric field $E$ is assumed along the $y$-axis, generated by the scalar potential $\Phi=-Ey$.

The Langevin white-noise forces $\zeta(\mathbf{r},\tau)$ are correlated through

$$s\left\langle\zeta^*(\mathbf{r},\tau)\zeta(\mathbf{r}',\tau')\right\rangle=2T\Gamma_0^{-1}\delta(\mathbf{r}-\mathbf{r}')\delta(\tau-\tau'), \tag{2}$$

where $s$ is the order parameter effective "thickness".

The fluctuation current density, averaged with respect to the noise, writes:

$$\mathbf{J}=\frac{ie^*\hbar}{2m^*}\left\langle\Psi^*\left(\nabla+i\frac{2\pi}{\Phi_0}\mathbf{A}\right)\Psi\right\rangle+c.c. \tag{3}$$

To solve Eq. (1), one uses the self-consistent Gaussian approximation which captures the most interesting fluctuations effects, in which the cubic term in the TDGL Eq. (1), $|\Psi|^2\Psi$, is replaced by a linear one $2\langle|\Psi|^2\rangle\Psi$. This results in a linear problem with a modified GL potential $\tilde{a}=a+2b\left\langle|\Psi|^2\right\rangle$, which implies a renormalized reduced temperature

$$\tilde{\varepsilon}=\varepsilon+\frac{2b\left\langle|\Psi|^2\right\rangle}{a_0}. \tag{4}$$

The average $\langle|\Psi|^2\rangle$ is to be determined, in principle, self-consistently together with the parameter $\tilde{\varepsilon}$.

It is hereafter more convenient to rescale the TDGL Eqs. (1) and (2) variables to the new ones: $x\to\xi x, y\to\xi y, \tau\to\tau\tau_{GL}, B\to hH_{c2}, E\to E_{GL}\mathcal{E}, \Psi\to\sqrt{2a_0/b}\varphi,\ \zeta\ \to\ (2a_0)^{3/2}/b^{1/2}\zeta$ with $\xi$ being the coherence length as a unit of length, $H_{c2}=\Phi_0/2\pi\xi^2$ as a unit of the magnetic field, $\tau_{GL}=\Gamma_0^{-1}\xi^2m^*/\hbar^2$as a unit of time, $E_{GL}=H_{c2}\xi/c\xi$ as a unit of electric field. Then the TDGL Eq. (1) becomes

$$(1+i\lambda)\left(\frac{\partial}{\partial\tau}+i\mathcal{E}y\right)\varphi-\frac{1}{2}\left[\left(\frac{\partial}{\partial x}-ihy\right)^2+\frac{\partial^2}{\partial y^2}\right]\varphi+\frac{1}{2}\tilde{\varepsilon}\varphi=\zeta. \tag{5}$$

The formal solution of this equation in zero electric field is

$$\varphi(\mathbf{r},\tau)=\int d\mathbf{r}'\int d\tau' G_0\left(\mathbf{r},\tau;\mathbf{r}',\tau'\right)\zeta\left(\mathbf{r}',\tau'\right). \tag{6}$$

The Green function takes a form

$$G_0\left(\mathbf{r},\mathbf{r}',\tau''\right)=C(\tau'')\theta\left(\tau''\right)\exp\left[\frac{ih}{2}X\left(y+y'\right)-\frac{X^2+Y^2}{2\beta}\right], \tag{7}$$

with $X=x-x', Y=y-y', \tau''=\tau-\tau'$. $\theta(\tau'')$ is the Heaviside step function. $C$ and $\beta$ are coefficients as following:

$$\beta=\frac{2}{h}\tanh\left(\frac{h}{2}\frac{\tau''}{1+i\lambda}\right), \tag{8}$$

$$C(\tau'')=\frac{h}{4\pi}\exp\left\{-\frac{\tilde{\varepsilon}}{2}\frac{\tau''}{1+i\lambda}\right\}csch\left(\frac{h}{2}\frac{\tau''}{1+i\lambda}\right). \tag{9}$$

Starting from Eqs. (6) and (7), we are able to calculate the density of Cooper pairs. The self-consistent Eq. (4) for the parameter $\tilde{\varepsilon}$ will therefore be taken a form after renormalization of the critical temperature

$$\tilde{\varepsilon}=\varepsilon-\frac{\omega_{2D}t}{\pi}\left(1+\lambda^2\right)\left[\psi\left(\frac{1}{2}+\frac{\tilde{\varepsilon}}{2h}\right)+\ln(2h)-\ln\left(1+\lambda^2\right)+\gamma_E\right], \tag{10}$$

where $\varepsilon=T/T_c-1$, and $\omega_{2D}=\sqrt{2Gi_{2D}}\pi$ where $Gi_{2D}=\frac{1}{2}(8e^2\kappa^2\xi^2T_c/c^2\hbar^2s)^2$ ($T_0$ is now replaced by $T_c$), $\psi$ is the polygamma function, $\gamma_E$ is Euler constant. The self-consistent equation is cutoff independent and obtained without need to assume anything about $\lambda$. For several HTSCs, $\lambda$ is a small parameter, reflecting the small Hall angle [4]. We shall therefore keep only the dominant zeroth order term in $\lambda$, the Eq. (10) takes a form

$$\tilde{\varepsilon}=\varepsilon-\frac{\omega_{2D}t}{\pi}\left[\psi\left(\frac{1}{2}+\frac{\tilde{\varepsilon}}{2h}\right)+\ln(2h)+\gamma_E\right]. \tag{11}$$

Eq. (11) matches the corresponding expressions already found in Ref. [28].