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Energy of magnetic moment of superconducting current in magnetic field

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ABSTRACT

The energy of magnetic moment of the persistent current circulating in superconducting loop in an externally produced magnetic field is not taken into account in the theory of quantization effects because of identification of the Hamiltonian with the energy. This identification misleads if, in accordance with the conservation law, the energy of a state is the energy expended for its creation. The energy of magnetic moment is deduced from a creation history of the current state in magnetic field both in the classical and quantum case. But taking this energy into account demolishes the agreement between theory and experiment. Impartial consideration of this problem discovers the contradiction both in theory and experiment.

1. Introduction

It is well known [1] that electric current I circulating clockwise or anticlockwise in a flat loop with a vector area $\mathbf{S}$ induces magnetic dipole moment equal $\mathbf{M_m} = I\mathbf{S}$. It is well known also [1] that magnetic moment in an externally produced magnetic field $\mathbf{B}$ has an energy equal $E_M = -\mathbf{M_m B}$. But this energy is not taken into account in the theory describing quantization effects in superconducting loop [2]. This discrepancy is particularly demonstrable in the case of persistent-current qubits [3] or flux qubits [4]. Flux qubits consist of a superconducting loop interrupted by either one or three Josephson junctions [5]. The two quantum states of flux qubit are persistent current I_p circulating in the loop clockwise and anticlockwise in an externally produced magnetic field $\mathbf{B}$ corresponding approximately the half $\mathbf{BS} = \Phi \approx (n+0.5)\Phi_0$ of the flux quantum $\Phi_0 = \pi\hbar/e$ inside the loop [5]. The qubit effective Hamiltonian are represented by the Pauli spin matrices σ_z, σ_x [4,6], that is

$$H_q = \epsilon\sigma_z - \Delta\sigma_x \tag{1}$$

as well as the Hamiltonian of spin – 1/2. The energy difference between two spin states of electron, for example, is the energy $\epsilon = \mu_B B_z$ of magnetic moment equal the Bohr magneton $\mu_B = -e\hbar/2m$ in external magnetic field B_z. This energy of flux qubit should be equal $|E_M| = M_m B_z = I_p\Phi$ when $\mathbf{B} = (0,0,B_z)$ and the flux qubit loop is in the flat $x-y$. But this energy is not take into account although the energy considered in the theory [4] $\epsilon = I_{pm}\Phi_0(\Phi/\Phi_0 - 1/2)$ (where I_{pm} is the maximum qubit persistent current) is much lower than the energy $|E_M| = M_m B_z = |I_p\Phi| \approx |I_p|\Phi_0/2$ near the half of the flux quantum $|\Phi/\Phi_0 - 1/2| \ll 1$.

2. Quantization effects in superconductors

The two states of flux qubit are assumed at $\Phi \approx (n+0.5)\Phi_0$ because of the requirement $\oint_l dl\nabla\varphi = 2\pi n$ that the complex pair wave function $\Psi = |\Psi|e^{i\varphi}$ must be single – valued at any point in a superconductor.

2.1. Quantization of angular momentum

Superconducting loop without Josephson junctions should also have such two states due to this requirement or the quantization of angular momentum of Cooper pair

$$m_p = \oint_l dlp/2\pi = \oint_l dl\hbar\nabla\varphi/2\pi = \hbar n \tag{2}$$

The idea of flux qubit presupposes the superposition of two macroscopic quantum states [5] assumed first by Anthony Leggett in the 1980s [7]. We will not consider the problem of superposition (described with the term $\Delta\sigma_x$ in Eq. (1)) assumed

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only in a superconducting loop interrupted by Josephson junctions. We take an interest in the energy difference ϵ of the term $\epsilon\sigma_z$ between two permitted states. Therefore superconducting loop without Josephson junctions will be considered first of all.

The relation

$$\mu_0 \oint_l dl\lambda_L^2 j + \Phi = n\Phi_0 \tag{3}$$

deduced from the requirement Eq. (2) can describe the Meissner effect, magnetic flux quantization and quantization of pair velocity or persistent current [8]. The Meissner effect i.e. the expulsion of magnetic flux Φ from the interior of a superconductor, discovered by Meissner and Ochsenfeld in 1933 [9], is observed in a bulk entire superconductor in which the wave function $\Psi = |\Psi|e^{i\varphi}$ has no singularity and therefore the quantum number $n = 0$ and, according to Eq. (3), $\Phi = n\Phi_0 = 0$ inside superconductor where the density of superconducting current $j = 0$. The flux quantization was observed first in 1961 [10] with the help of measurements of magnetic flux trapped in hollow superconducting cylinder the wall width w of which is larger $w \gg \lambda_L$ than the London penetration depth $\lambda_L = (m/\mu_0 q^2 n_s)^{0.5} = \lambda_L(0)(1 - T/T_c)^{-1/2}$ [11] ($\lambda_L(0) \approx 50\,\mathrm{nm} = 5 \times 10^{-8}$ m for most superconductors [2]). The current density $j \approx 0$ along a contour l inside superconducting region in this case of strong screening and therefore $\Phi \approx n\Phi_0$ according to Eq. (3).

The quantization of the persistent current

$$\frac{\lambda_L^2}{s}\mu_0 l I_p = (n\Phi_0 - \Phi) \tag{4}$$

or velocity $\oint_l dl v = (2\pi\hbar/m)(n - \Phi/\Phi_0)$ is observed in the case of weak screening, for example in a loop with section $s \ll \lambda_L^2$. The kinetic inductance $L_k \approx (\lambda_L^2/s)\mu_0 l$ exceeds in this case the magnetic inductance $L \approx \mu_0 l$ and one can always neglect the magnetic flux $\Delta\Phi_I = LI_p$ induced with the current I_p for a sufficiently thin superconductor [2]. Therefore the magnetic flux $\Phi = BS + LI_p$ equals approximately the one $\Phi \approx BS$ of externally produced magnetic field B. Quantization effect in the weak screening limit was observed first by Little and Parks [12] at measurements of the resistance of thin cylinder in the temperature region corresponding to its superconducting resistive transition. Later on quantum periodicity of other quantities were observed: ring resistance [13,14], magnetic susceptibility [15], critical current [16] and dc voltage measured on segments of asymmetric rings [13,14,17–20]. Superconducting ring according to Eq. (4), as well as flux qubit, has at $\Phi = (n' + 0.5)\Phi_0$ the two permitted current states $I_{pm} = (n\Phi_0 - \Phi)/L_k = -0.5\Phi_0/L_k$ when $n = n'$ and $I_{pm} = +0.5\Phi_0/L_k$ when $n = n' + 1$.

2.2. Energy and Hamiltonian

The energy difference ϵ of these states is deduced from the Hamiltonian

$$H = \frac{1}{2m}\sum_a [-i\hbar\nabla_a - qA(r_a)]^2 + U \tag{5}$$

used for description of quantization effects in superconductors as far back as 1961 [21]. According to this Hamiltonian the energy of a sufficiently thin superconducting loop with homogeneous Cooper pair density $|\Psi|^2 = n_s$ should equal $\int_V dV\Psi * H\Psi = \int_V dV|\Psi|^2[(1/2m)(p - qA)^2 + U] = \int_l dl s n_s \frac{mv^2}{2} + \int_V dVn_s U$. The potential energy $\int_V dVn_s U$ does not depend on magnetic flux Φ and is not considered in the theory of quantization. The kinetic energy of Cooper pairs

$$E_k = \oint_l dl s n_s \frac{mv^2}{2} = \frac{I_p}{q}\oint_l dl \frac{mv}{2} = L_k I_p^2/2 \tag{6}$$

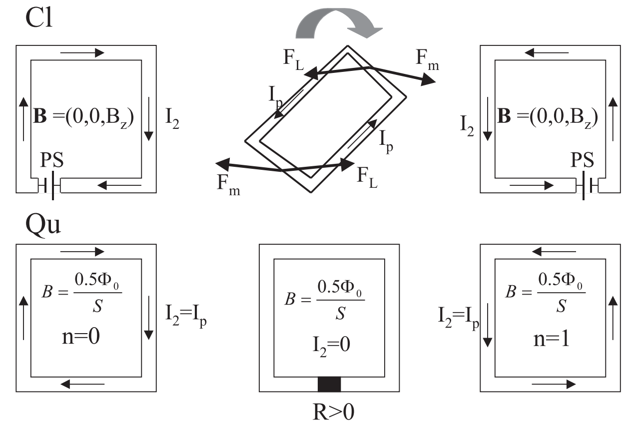

Fig. 1. Cl: The electric current I_2 circulating in the loop induces magnetic dipole moment $\mathbf{M_m} = I_2\mathbf{S}$. Moment $\tau = \mathbf{M_m} \times \mathbf{B}$ of force F_L acts on this loop in magnetic field $\mathbf{B}$. The moment of mechanical force F_m should be applied and the energy $E_M = \int_0^\pi d\theta M_m B_z \sin\theta = 2M_m B_z$ should be expended in order to overturn the loop when the constant value of the current I_2 is maintained with the help of power source PS. Clockwise current changes into anticlockwise current relatively the B_z direction after this turning-over. Qu: The direction of the persistent current in superconducting loop changes with quantum number n change as a result of the transition of a loop segment in normal state (black) with a non-zero resistance $R > 0$ and posterior retrieval it in superconducting state.

does not depend on direction of the velocity v or the current I_p. Thus, two permitted state n and $n+1$ with different angular momentum have the same energy $\int_V dV\Psi^* H\Psi = L_k I_{pm}^2/2 + \int_V dVUn_s = I_{pm}0.5\Phi_0/2 + \int_V dVUn_s$ at $\Phi = (n + 0.5)\Phi_0$ and the energy difference $\epsilon = 0$ according to the canonical Hamiltonian Eq. (5).

But we know that the energy of two states having different magnetic moment in non-zero magnetic field should be different. Clockwise electric current I_p can be obtained from anticlockwise current I_p with the help of the turning-over of the loop, Fig. 1Cl. It is well known that we should expand the energy $E_M = \int_0^\pi d\theta M_m B_z \sin\theta = 2M_m B_z$ in order the rotate the magnetic dipole moment $\mathbf{M_m} = I_{pm}\mathbf{S}$ in magnetic field B_z [1]. "But when we go over to the Hamiltonian formalism by the standard 'canonical' procedure, the total Hamiltonian $(1/2m)(p - qA)^2$ turns out to be just the kinetic energy $mv^2/2$! Where has the 'magnetic' energy gone?" [22].

3. Identification of Hamiltonian with energy misleads

Anthony Leggett has surmised soundly that "Perhaps our naive tendency to identify the Hamiltonian with the 'energy' is (as in some cases involving time-dependent forces) misleading?" [22].

3.1. Energy expended for the current in magnetic field

Indeed, the energy of electric current circulating in a perfect conductor deduced from the classical Hamiltonian (16.10) in [23]

$$H = \frac{1}{2m}(p - qA)^2 + q\phi \tag{7}$$

turns out to be just the kinetic energy $\int_V dVn_q H = \int_V dVn_q(1/2m)(p - qA)^2 = \int_l dl s n_s mv^2/2 = L_k I^2/2$ at weak screening $L \ll L_k$ as well as in the quantum case Eq. (6). This energy should be expended

$$\int_t dtIV = \int_t dtI(L_k + L)\frac{dI}{dt} = \frac{(L_k + L)I_2^2}{2} \approx \frac{L_k I_2^2}{2} \tag{8}$$