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Fracture problem of a nonhomogeneous high temperature superconductor slab based on real fundamental solutions

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ABSTRACT

To analyze the fracture problem of the nonhomogeneous high temperature superconductor (HTS) slab under electromagnetic force, we derive the real fundamental solutions based on eigenvalue and eigenvector analyses. The superconductor E-J constitutive law is characterized by the Bean model where the critical current density is independent of the flux density. Fracture analysis is performed by the methods of singular integral equations which are solved numerically by Lobatto–Chybeshev collocation method. Numerical results of the stress intensity factor (SIF) are obtained. Moreover, the crack opening displacement (COD) can be obtained by numerical integration dislocation density functions. The effects of the thickness ratio, HTS material nonhomogeneous parameters, applied magnetic field and critical current density on SIF and COD are discussed. The present work could theoretically provide quantitative predictions of the fracture mechanism of the nonhomogeneous HTS.

1. Introduction

In recent years, high temperature superconductors (HTS) have been widely applied to magnetic bearings, flywheel storage energy systems, trapped field magnet due to their unique properties of high critical current density and trapped field [1]. In practical applications, especially to the trapped field magnet, the superconductors are often simultaneously subjected to such forces as electromagnetic force and thermal stress, which may lead to their fracture [2,3]. In melt-processed superconductors, micro-cracks are an important phenomenon because bulk superconductors are brittle ceramic in the form of pseudo-single crystal. In a large single-grain bulk superconductor, it has been reported that cracks occur when the applied field is decreased from 10 to 0 T at 50 K [2]. In addition, it was found that the cracks of the bulk superconductors could limit the trapping ability at lower temperatures [4]. When a cracked superconductor is subjected to a large electromagnetic force, the high stress concentration may initiate crack growth and eventually lead to fracture.

The fracture analysis of HTS has become a new but important field in the fracture mechanics. In the HTS fracture mechanics, an accurate evaluation of stress intensity factor (SIF) is crucial for both static magnetic field and alternating magnetic field as it can be used to investigate crack initiation and propagation. In general, the fracture of HTS is induced by the electromagnetic force which is flux-pinning-induced magnetoelasticity phenomenon. The flux-pinning-induced magnetostriction phenomenon was firstly described by Ikuta et al. [5,6]. Çelebi and İnanir reported normal-state contribution on the pinning induced magnetostriction [7,8]. The mechanical stress caused by flux pinning on superconductor was investigated theoretically by Johansen and Yong [9,10]. Yong et al. (2012a) quantitatively illustrated the effects of coupling parameters on magnetostriction and magnetization [11]. An accurate treatment of such problems often requires a realistic modeling of the complex couplings between different phenomena involved in the real magneto–thermo-mechanical behavior of HTS. For the design and application of superconductors in engineering, it is significantly important to study the fracture behavior in a magnetic field [12]. Therefore, SIFs are the important fracture parameters in understanding and predicting fracture behavior of a cracked superconductor. There are some existing literatures dealing with various fracture problems of the homogeneity superconductor. Zhou et al. [12] firstly studied the crack problem for a long rectangular slab of superconductor under an electromagnetic force. Moreover, Gao et al. studied the crack-inclusion problem for a long rectangular slab of superconductor and two collinear crack problems [13,14]. Yong et al. (2012) investigated the crack problem for a thin superconducting strip in a perpendicular magnetic field [15]. Recently, Gao et al. studied the dynamic fracture problem of superconductor

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under an alternating magnetic field [16]. These fracture analyses are conducted based on the assumption that superconductors are isotropic homogeneity material. However, superconductors are in fact nonhomogeneous material with different material properties in *a*,*b* and *c* directions, as is shown in Young's modulus, thermal expansion coefficients, and thermal conductivity [17,18]. Therefore, the fracture analysis of nonhomogeneous HTS is a key issue in superconductor crack problems. Feng et al. have studied the crack problem for a functionally graded thin superconducting film [19–21].

Generally, the mode-I crack problems are solved analytically in complex domain because in-plane governing equations have complex fundamental solutions. For example, Ma et al. studied the fracture behaviors of nonhomogeneous elastic strip using integral transform and singular integral equation methods [22]. However, the mode I crack problems can be analyzed in real domain. Li firstly used the real fundamental solutions to study the mode-I crack problem [23–25]. Govorukha investigated the interface crack problem for piezoelectric materials [26,27]. Zhou studied the thermal fracture problem of functionally graded coating-substrate structure by the real fundamental solutions [28]. The real fundamental solutions have an advantage over complex fundamental solutions, because the real analyses are more straightforward than the complex analyses. For the mode I cracks, the governing equations are a system of coupled partial differential equations and the characteristic equation generally has complex eigenvalues and eigenvectors. Most existing literatures used these complex eigenvalues and eigenvectors to construct complex fundamental solutions, and subsequently, directly performed fracture mechanics derivation based on the complex fundamental solutions. Because of the complexity of the corresponding complex analysis, one may encounter difficulties in handling the mixed boundary value conditions. However, if we can construct real fundamental solutions, the subsequent fracture analysis would become more straightforward, because as is well known, the real analysis is much simpler than the complex analysis.

In this paper, a new real fundamental solutions method is proposed to study the mode-I fracture problem of the nonhomogeneous HTS under electromagnetic force. The crack problem is analyzed in the real number field by recasting the solutions in real form and by the methods of Cauchy singular integral equation and Lobatto–Chybeshev collocation method. Moreover, this paper firstly investigates the crack opening displacement problem of HTS under electromagnetic force by numerical integration dislocation density functions. The analysis is based on the following assumptions: (1) the demagnetization effects are neglected and (2) the magnetic behavior can be described by the critical state model.

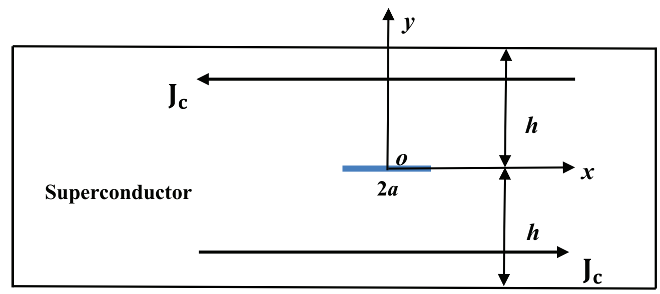

Fig. 2. Illustration of the 2D configuration of a cracked nonhomogeneous superconductor.

2. Problem formulation

Illustrated in Fig. 1 is a nonhomogeneous HTS with a crack placed in a magnetic field oriented parallel to the *z* direction and the crack lies in the *x*–*y* plane. The crack of length $2a$ is parallel to the upper and lower boundaries and the distance between the crack and the boundaries is h as is shown in Fig. 2. A rectangular coordinate system is established with the rightward *x* axis parallel to the crack line and the upward *y* axis through the crack center. The superconductor slab is assumed sufficiently long in the *z* direction, and then, the demagnetization effects can be negligible.

The electromagnetic forces arising from flux-pinning are body forces. To analyze the effects of the electromagnetic force, it is necessary to know the distribution of the flux density inside the superconductor. For simplicity, we adopt the Bean model where the critical current density is independent of the flux density [29]. When the applied magnetic field is increased above the full penetration value, the critical current fills the entire superconductor. However, as the applied field starts to decrease from its maximum value, the direction of the critical current reverses in the outer part of the slab. Therefore, the total forces applied on the crack surfaces may be tensile forces.

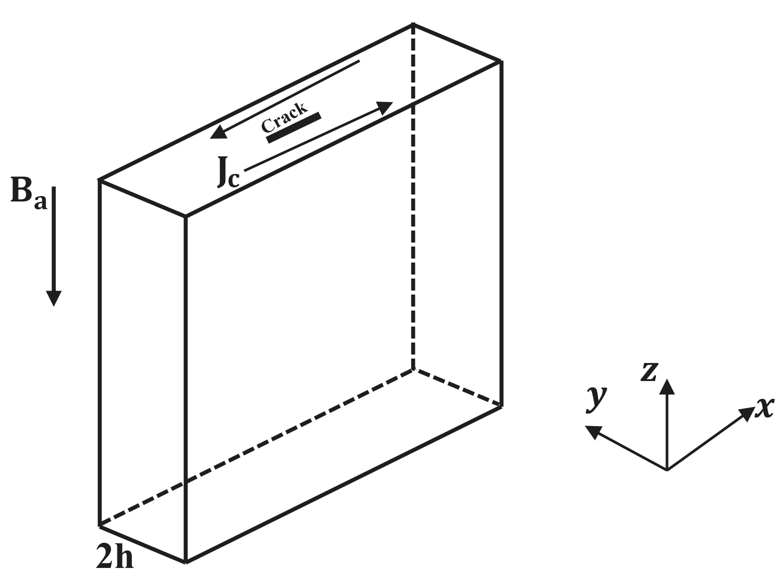

Fig. 1. Schematic of long rectangle nonhomogeneous superconductor under magnetic field.