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Scaling law and general expression for transport ac loss of a rectangular thin strip with power-law $E(J)$ relation

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ABSTRACT

Transport ac loss Q of a superconducting rectangular thin strip obeying a power-law relation $E\propto J^n$ as a function of current amplitude I_m may be, following Norris, expressed by normalized quantities as $q(i_m)$. A scaling law is deduced that if $I_c f$, I_c and f being the critical current and frequency, is multiplied by a positive constant C, then i_m and q_m are multiplied by $C^{1/(n-1)}$ and $C^{2/(n-1)}$, respectively. Based on this scaling law and the well-known Norris formula, the general function of $q(i_m, n, f)$ is obtained graphically or analytically for any practical purpose, after accurate numerical computations on a set of $q(i_m)$ at several values of n and a fixed value of f.

1. Introduction

Soon after Bean assumed in the critical-state (CS) model a constant critical current density J_c to calculate the magnetization curve of a hard superconducting cylinder [1], London reported his result of the transport ac loss of a hard superconducting cylinder derived from the same assumption [2]. For a long cylinder of radius a carrying a transport current

$$I(t) = I_m \sin 2\pi ft, \tag{1}$$

London obtained a CS relation between the ac loss Q per cycle per unit length and I_m as

$$q = (2 - i_m)i_m + 2(1 - i_m)\ln(1 - i_m), \tag{2}$$

where

$$q \equiv 2\pi Q/\mu_0 I_c^2, \;\; i_m \equiv I_m/I_c, \tag{3}$$

$I_c = \pi a^2 J_c$ being the critical current. The validity of this formula was later extended by Norris to a bar of elliptical cross-section with any values of semi-axes a and b [3]. Eq. (2) for thin elliptical tapes is relevant to 1G HTS (high-temperature superconductor) tapes and has been well verified by both numerical calculations and analytical derivation [4,5]. Norris also derived a formula for a rectangular thin strip [3], and with the same normalization as in Eq. (3) it is expressed by

$$q = 2[(1 + i_m)\ln(1 + i_m) + (1 - i_m)\ln(1 - i_m) - i_m^2]. \tag{4}$$

This CS equation is more relevant to the 2G HTS tapes, for which a HTS film is epitaxially deposited on a metallic substrate with a number of buffer layers in between. The current-density and field distributions corresponding to this case have been calculated by Brandt and Indenbom [6].

It has been shown that the measured q vs i_m curves of HTS tapes are roughly located around the modeling curves calculated from Eqs. (2) and (4), which means that the CS model is basically valid for such tapes. However, this comparison between experimental and modeling results is somewhat ambiguous, since the relations between current density J and electrical field E in the actual HTS tapes and in the CS model are significantly different. In the CS model, $|J|$ cannot be larger than J_c, $E = 0$ occurs when $|J| < J_c$, and a finite E appears when $|J| = J_c$. As a result, I_c for a tape is a fixed value and the ac loss Q is f independent. On the other hand, in transport current–voltage (I-V) measurements of most HTS tapes, I changes with V in the full penetration region following roughly a power law (PL), $V\propto I^n$, so that I_c has to be defined as that when $E = V/l = E_c$, where l is the distance between both voltage taps and criterion

$$E_c = 10^{-4}\mathrm{V/m} \tag{5}$$

is routinely defined. Related to this, ac loss Q is intrinsically f dependent.

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The PL *I-V* curve comes from a local PL $E(J)$ relation, which is expressed as [7–9]

$$\mathbf{E} = (E_c/J_c)|J/J_c|^{n-1}\mathbf{J}. \tag{6}$$

In fact, the CS model may be regarded as a limit case of the PL $E(J)$ relation with $n \to \infty$, so that the $q(i_m)$ relations expressed by Eqs. (2) and (4) are the PL model $q(i_m)$ at $n \to \infty$.

When studying magnetic properties of superconductor discs, cylinders, and rings with a PL $E(J)$ relation, Brandt has proposed a scaling law as follows [9]. When one changes the time unit by an arbitrary constant factor of C and the current and field units by a factor of $C^{1/(n-1)}$, then the equations of motion for the current density are invariant; i.e., the same solutions result for the scaled quantities. For the ac case if the applied field is $H_a(t) = H_m \sin 2\pi ft$, the shape of the hysteretic magnetization curve $M(H_a)$ remains unchanged if one increases f by a factor of C and H_m by a factor of $C^{1/(n-1)}$. Although the scaling law has been proved explicitly only for superconductors of particular shapes, it is thought to be geometry independent without further arguments [9]. Using this scaling law, a firm base is provided for the ac susceptibility technique of J_c determination of advanced HTS samples [10].

The ac loss Q and $E(I)$ curves of a cylinder obeying Eq. (6) have been calculated by Chen and Gu [11,12] with a transport scaling law proposed. This law has been deduced for the case of cylinder and verified numerically for strip [11,13]. Different from the above scaling law for ac magnetization, where f is multiplied by C for any given superconductor, in the transport scaling law the product I_cf is multiplied by C, so that the ac loss of any pair of different superconducting cylinders or strips may be mutually scaled to each other, and a set of computation results for cylinders or strips with arbitrarily chosen values of I_c and f may be used for any other values of I_c and f. In particular, $q(i_m)$ curve at any value of f may be scaled to one at critical frequency f_c that has a common point with the CS curve expressed by Eq. (2) or (4) at $i_m = 1$. Computing a set of such $q(i_m)$ curves scaled to f_c at several values of n as a base, $q(i_m)$ for any values of I_c, f, and n can be obtained by interpolations and scaling.

Since this transport scaling law for the studied strips has not been proved analytically in [13], we will do it in Section 2. We will improve the computations done in [13] to obtain more accurately the basic set of $q(i_m)$ at f_c and give its general expression, from which $q(i_m)$ at $n > 5$ and any values of f can be obtained, in Section 3. The PL current density and electrical field profiles are shown and some applications of the scaling law and the general expression of the basic set of $q(i_m)$ are described in Section 4 before the conclusions, which are presented in Section 5.

2. Transport scaling law

The studied rectangular thin strip is placed along the z axis located at $|y| \le d/2$ and $|x| \le a \gg d$. It is characterized by a PL $E(J)$ relation as Eq. (6). We calculate Q at different values of I_c, I_m, n, and f by applying the procedure described in [13–15]. Defining the surface current density $K = Jd$, Eq. (6) is written

$$\mathbf{E} = (E_c/K_c)|K/K_c|^{n-1}\mathbf{K} = \rho(K)\mathbf{K}, \tag{7}$$

where $K_c = I_c/2a$ is the critical surface current density. Dividing the width $2a$ into N equal elements, each centered at $x_i (i = 1, 2, \ldots, N)$, the computation is started by calculating a matrix of components [16]

$$\begin{aligned} Q_{ij} &= \ln|x_i - x_j|/2\pi \quad (j \ne i) \\ &= \ln(a/\pi N)/2\pi \quad (j = i). \end{aligned} \tag{8}$$

This matrix is defined for converting the surface current density $K_j (j = 1, 2, \ldots, N)$ to vector potential $A_i (i = 1, 2, \ldots, N)$ in the London gauge (Coulomb gauge) by

$$A_i = -\frac{\mu_0 2a}{N}\sum_{i=1}^{N} Q_{ij}K_j, \tag{9}$$

and the components of its reciprocal Q_{ij}^{-1} are used for calculating K_i by solving numerically a system of equations for $i = 1, 2, \ldots, N$,

$$\frac{dK_i}{dt} = \frac{N}{2\mu_0 a}\sum_{j=1}^{N} Q_{ij}^{-1}[K_j\rho(K_j) - E_a], \tag{10}$$

$$\frac{2a}{N}\sum_{i=1}^{N} K_i = I_m \sin 2\pi ft, \tag{11}$$

where nonlinear resistivity $\rho(K_j)$ is defined in Eq. (7) and E_a is mathematically an integration constant with respect to position and physically an electrical field energetically applied by the power supply, which will be discussed elsewhere, and the aim of numerical computation is to find a proper function of $E_a(t)$ to satisfy Eq. (11) with sufficient accuracy.

Having obtained $K_i(t)$ for $i = 1, 2, \ldots, N$, the loss power per unit length at time t is calculated by

$$P(t) = \sum_{i=1}^{N} K_i^2 \rho(K_i)\frac{2a}{N}. \tag{12}$$

The final loss per cycle per unit length is calculated by

$$Q = \int_{mT}^{(m+1)T} P(t)dt, \tag{13}$$

where $m + 1 \ge 2$ is the number of periods having calculated.

In order to prove the scaling law, dimensionless quantities of t and K are defined as

$$\tau = 2\pi ft, \tag{14}$$

$$\kappa = K/K_c, \tag{15}$$

so that Eqs. (10), (11), and (13) may be written as

$$\frac{d\kappa_i(\tau)}{d\tau} = \frac{NE_c}{2\pi\mu_0 fI_c}\sum_{j=1}^{N} Q_{ij}^{-1}[\kappa_j(\tau)|\kappa_j(\tau)|^{n-1} - E_a(\tau)/E_c], \tag{16}$$

$$i_m \sin\tau = \frac{1}{N}\sum_{i=1}^{N}\kappa_i(\tau), \tag{17}$$

$$\begin{aligned} q &= \frac{2\pi Q}{\mu_0 I_c^2} = \frac{1}{\mu_0 I_c^2 f}\int_{m2\pi}^{(m+1)2\pi} P(\tau)d\tau \\ &= \frac{E_c}{\mu_0 N I_c f}\int_{m2\pi}^{(m+1)2\pi}\sum_{i=1}^{N}|\kappa_i(\tau)|^{n+1}d\tau. \end{aligned} \tag{18}$$

The scaling law may be deduced from Eqs. (16)–(18) as follows. Since $Q_{ij} (i, j = 1, 2, \ldots, N)$ are constants for any given values of a and N as expressed by Eq. (8), the solutions $\kappa_i(\tau) (i = 1, 2, \ldots, N)$ and $E_a(\tau)/E_c$ of Eqs. (16) and (17) for any given value of n are determined by I_cf/E_c only. It can be found by examining Eq. (16) that if I_cf/E_c is multiplied by a constant C, then the solutions $\kappa_i(\tau)$ and $E_a(\tau)/E_c$ are multiplied by $C^{1/(n-1)}$ and $C^{n/(n-1)}$, respectively, so that i_m and q in Eqs. (17) and (18) are multiplied by $C^{1/(n-1)}$ and $C^{2/(n-1)}$, respectively. If a fixed E_c is expressed by Eq. (5) and subscripts 1 and 2 are used for mutually scaled two cases, the scaling law may be stated as that if

$$(I_cf)_2/(I_cf)_1 = C, \tag{19}$$

then

$$i_{m,2}/i_{m,1} = C^{1/(n-1)}, \quad q_2/q_1 = C^{2/(n-1)}. \tag{20}$$

Since similar derivations may be applied to long superconductors of any cross-section, the transport scaling law is generally valid.