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Simulation of magnetization process of Pure-type superconductor magnet undulator based on T-method

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ABSTRACT

For the next generation Free Electron Laser, Pure-type undulator made of high Tc superconductors (HTSs) was considered to achieve a small size and high intensity magnetic field undulator. In general, it is very difficult to adjust the undulator magnet alignment after the HTS magnetization since the entire undulator is installed inside a cryostat. The appropriate HTS alignment has to be determined in the design stage. This paper presents the development of a numerical simulation code for magnetization process of the Pure-type HTS undulator to assist the design of the optimal size and alignment of the HTS magnets.

1. Introduction

As one of challenging technologies in the next generation Free Electron Laser (FEL), application of high-Tc superconductors (HTS) for undulator magnets has been proposed to obtain very strong magnetic field [1–3]. Since the HTSs for the undulator have to be magnetized all together in a cryostat, it is not easy to create sinusoidal magnetic field along the electron trajectory. The "Pure-type" HTS undulator was proposed as one of possible HTS magnet arrays for undulators [2]. In the design of the HTS undulator, it is very important to understand and to predict the magnetization process of the HTSs. The appropriate size and alignment of the HTS undulator magnets are determined to create the exactly sinusoidal magnetic field. The numerical simulation of the magnetization process plays a very important role to design the HTS magnet.

This paper presents a numerical simulation of the magnetization process of the Pure-type HTS undulator based on the current vector potential method (T-method), combining with the critical state model for the shielding current in the HTS. The simulation results are compared with experimental measurements of the trapped fields in the HTS. Appropriate values of critical current and external magnetic field are considered by using the developed code. A simulation of a single electron motion is shown to confirm the calculated undulator magnetic field.

2. Pure-type superconducting undulator

Fig. 1(a) shows an overview of the Pure-type HTS undulator made of three HTSs [2]. In the field-cooled magnetization process, external magnetic field is first applied to the HTS, and the HTS is cooled to be the superconducting state. During the external magnet field is gradually reduced to $B_{\min} < 0$, as the bias field (Fig. 1(b)), shielding currents are induced and the field is trapped in the HTS as shown in Fig. 1(c). The alternating vertical magnetic field is created on the HTS by these shielding currents. The FEL undulator magnetic field is composed by superposition of the shielding current magnetic field and the externally bias field.

3. Simulation of magnetization process of Pure-type HTS undulator based on T-method

The numerical simulation has very important role to understand the HTS magnetization process and the design of the undulator magnet array. In this work, we here use the current vector potential method (T-method) [4–7] for the simulation of the induced shielding current combing with the critical state model. The shielding current $\mathbf{J}$ induced in the HTS is expressed by using the current vector potential $\mathbf{T}$ defined by $\mathbf{J} = \nabla \times \mathbf{T}$ and the governing equation for $\mathbf{T}$ is the following integro-differential equation,

$$\nabla \times \frac{1}{\sigma} \nabla \times \mathbf{T} - \mu_0 \frac{\partial \mathbf{T}}{\partial t} - \frac{\mu_0}{4\pi} \int_s \frac{\partial \mathbf{T} \cdot \mathbf{n}}{\partial t} \nabla' \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) dS' = \frac{\partial \mathbf{B}_0}{\partial t} \quad (1)$$

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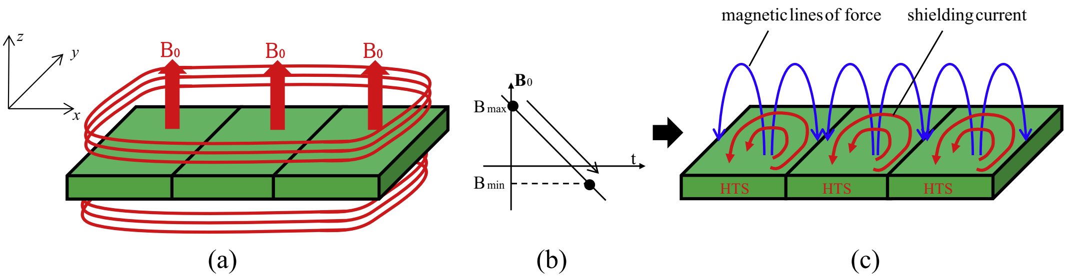

(a) (b) (c)

Fig. 1. Overview of Pure-type undulator.

where σ is conductivity, μ_0 is permeability, S is surface of the HTS, $\mathbf{n}$ is a unit normal vector on S, and $\mathbf{B_0}$ is externally applied magnetic field. The current vector potential has to satisfy the following boundary and gauge conditions respectively,

$$\mathbf{T}\times\mathbf{n}=0 \quad \text{on } S, \tag{2}$$

$$\nabla\cdot\mathbf{T}=0 \quad \text{in domain.} \tag{3}$$

We here assume that the shielding current is induced in x–y horizontal plane, since the bulk HTS has anisotropic critical current. Therefore, the HTS can be expressed by the thin-plate multi-layers model (Fig. 2), and the current vector potential has only z-component. Ohm's law is modified to the following critical state model [4–6] for describing the shielding current behavior in the HTS,

$$\begin{cases} \mathbf{J}=J_c\frac{\mathbf{E}}{|\mathbf{E}|} & \text{if}\,|\mathbf{E}|\neq 0, \\ \frac{\partial\mathbf{J}}{\partial t}=0 & \text{if}\,|\mathbf{E}|=0. \end{cases} \tag{4}$$

That is, the electric field $\mathbf{E}$ is induced in a local region by change of the magnetic field, shielding currents with the critical current density J_c are obtained. If there is no electric field by shielding effect, situation of the currents is not changed. Though the critical current density J_c has a dependence on the magnetic field, the Bean model (J_c = constant) is applied to the present analysis since the critical current density is almost constant in the low cryogenic temperature.

To implement the critical state model (4) into the T-method in (1), we use the following artificial conductivity scheme [5,6]. At first, conductivity in all elements is set to very large value, e.g. 10^{13} [1/Ωm], assuming the superconductor is a very good conductor. If current over the critical current density J_c is obtained, the conductivity of the element is corrected as follows,

$$\begin{cases} \sigma_{\text{new}}=\sigma_{\text{old}}\frac{J_c}{J} & \text{if } J>J_c, \\ \sigma_{\text{new}}=\sigma_{\text{old}} & \text{if } J\leqslant J_c. \end{cases} \tag{5}$$

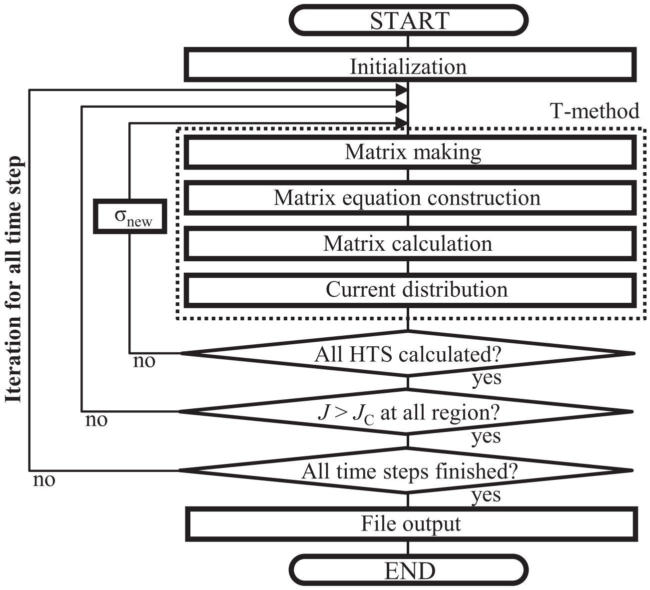

Fig. 3. Flowchart of simulation of magnetization process.

In each time step of the T-method in (1), conductivity distribution is modified by (5) so that the shielding current $\mathbf{J}$ is smaller than the critical current J_c in all region of the HTS.

A flowchart of the simulations of the magnetization process of the Pure-type HTS undulator based on the T-method is indicated in Fig. 3. The T-method calculations of the shielding current $\mathbf{J}$ are carried out for the individual HTS and the conductivity distribution σ is modified according to (5) in each time step. When the condition $|\mathbf{J}|<J_c$ is satisfied in all HTSs, the simulation proceeds to the next time step. This calculation is repeated from the initial time with $\mathbf{B_0}=B_{\max}$ to the final time with $\mathbf{B_0}=B_{\min}$(Fig. 1). We carry out numerical discretization of the Pure-type HTS undulator by

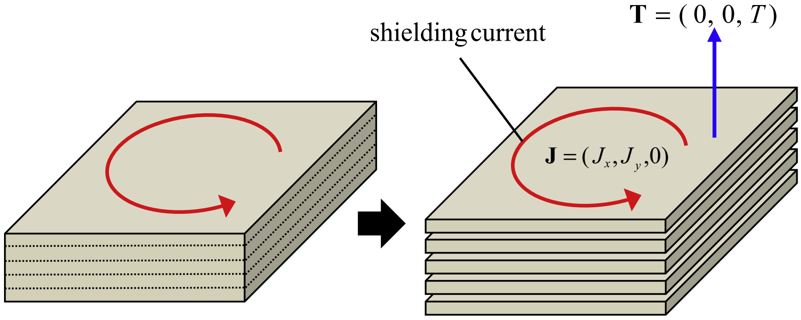

Fig. 2. Thin plate multi-layers modeling of HTS.

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