Physica C 515 (2015) 7–14

Contents lists available at ScienceDirect

Physica C

journal homepage: www.elsevier.com/locate/physc

Superconducting slab in an antisymmetric magnetic field: Vortex–antivortex dynamics

G. Carapella [a,*], P. Sabatino [a], M. Gombos [b]

[a] CNR-SPIN and Dipartimento di Fisica "E. R. Caianiello", Università degli Studi di Salerno, I-84084 Fisciano, Sa, Italy
[b] CNR-IMM UOS Napoli, Italy

ARTICLE INFO

Article history:
Received 11 March 2015
Received in revised form 2 May 2015
Accepted 5 May 2015
Available online 14 May 2015

Keywords:
Vortex antivortex
Ginzburg Landau
Rectifier
Surface energy barrier

ABSTRACT

When a type II superconductor with slab geometry is subjected to a magnetic field which is antisymmetric with respect to the middle of the slab the induced vortex matter consists of vortex–antivortex pairs or double kinks. These double kinks and their role in the generation of a considerable asymmetry in the critical current of the slab are addressed here both numerically, in the framework of time dependent Ginzburg Landau model, and semi-analytically, using the concept of surface energy barriers for flux entry and flux exit.

1. Introduction

Superconducting rectifiers [1–16] are based on superconducting devices that exhibit asymmetric voltage–current [$V(I)$] curves. Asymmetric $V(I)$ curves have been reported [9–15] in Ferromagnet–Superconductor hybrids on the micron scale. The asymmetry in the critical [17] currents has also been predicted and observed in single thin superconducting strips with different edges [18,19] (asymmetric surface barriers), in mesoscopic strips with turns [20–23] (current crowding) subjected to homogeneous magnetic field applied perpendicular to the strip, and in curved mesoscopic [24] superconducting thin strips in parallel magnetic field. Very recently, we have experimentally demonstrated [25] that a marked asymmetry in the critical currents can be also exhibited by a single superconducting slab having plano-convex cross section when subjected to a homogeneous magnetic field applied parallel to the substrate.

Here we theoretically address the physics of a type II superconductor with slab geometry [17] subjected to an *antisymmetric* magnetic field applied parallel to the slab and perpendicular to the transport current density. Being the applied field antisymmetric with respect to middle of the slab, in this system the vortex matter [26] always consists of vortex–antivortex pairs or double kinks that play a significant role in the generation of an asymmetry in the critical current that makes the addressed system possibly interesting as a superconducting rectifier. The double kink mechanism is addressed here both numerically, in the framework of time-dependent Ginzburg Landau model [17], and semi-analytically, in the framework of a simple extension of surface barrier models.

The analytical investigation of energy barriers for flux entry and flux exit in superconducting slabs or strips subjected to an homogeneous applied magnetic field, also in the presence of a transport current, is a subject reported [17,21,27–33] in the literature. However, to our knowledge, the analysis of surface barriers specialized to the *slab geometry* in the presence of both transport current and *inhomogeneous* magnetic field, is a subject not addressed before in the literature and it represents the main novelty of the present work. Moreover, numerical investigations of the magneto-transport properties of superconductors with slab geometry in the specific framework of Ginzburg Landau model have been reported [34–37,31,38] for the case of homogeneous applied field, whereas in the present work numerical simulations for the different case of an *inhomogeneous* applied field are reported.

The work is organized as follows. In Section 2 the transport properties of the superconducting slab in inhomogeneous magnetic field are numerically addressed in the framework of time-dependent Ginzburg Landau model [17], that is reliably used [34–37,39–41] whenever direct computation of voltage–current curves of a type II superconductor in the presence of magnetic field is needed. In Section 3 we give an approximated analytical description of the surface barriers and the forces experienced by

* Corresponding author. Tel.: +39 089968221; fax: +39 089969658.
E-mail address: giocar@sa.infn.it (G. Carapella).

http://dx.doi.org/10.1016/j.physc.2015.05.001
0921-4534/© 2015 Elsevier B.V. All rights reserved.

the double kinks and their role in the generation of asymmetric transport properties of the slab. A brief summary of main results is given in Section 4.

2. Model and numerical results

The transport properties of the superconducting slab are numerically investigated using the time-dependent Ginzburg–Landau (TDGL) model [34–37,39,41] that reads:

$$u\frac{\partial\psi}{\partial t} = (\nabla - i\mathbf{A})^2\psi + \left(1 - T - |\psi|^2\right)\psi, \quad (1)$$

$$\frac{\partial \mathbf{A}}{\partial t} = \frac{1}{2i}(\psi^*\nabla\psi - \psi\nabla\psi^*) - |\psi|^2\mathbf{A} - \kappa^2\nabla\times(\nabla\times\mathbf{A} - \mathbf{H}). \quad (2)$$

Here $\psi = \psi(x,y,z)$ is the complex order parameter, $\mathbf{A} = (A_x, A_y, A_z)$ is the vector potential, $\mathbf{H}$ is the applied magnetic field, T is the temperature, κ is the Ginzburg–Landau parameter and the coefficient $u = 5.79$ controls the relaxation of ψ. All physical quantities are measured in dimensionless units: the coordinates are in units of the coherence length $\xi(0) = \sqrt{\pi\hbar D/8k_BT_c}$, with T_c the critical temperature, and D is the diffusion constant. Temperature is in units of T_c. Time is measured in units of the relaxation time $\tau(0) = 4\pi\sigma_n\lambda(0)^2/c^2$ (σ_n is the normal-state conductivity, $\lambda(0) = \kappa\xi(0)$ the magnetic field penetration depth, with κ the G–L parameter). The order parameter is in units of $\Delta(0) = 4k_BT_c\sqrt{u}/\pi$, i.e., the superconducting gap at $T = 0$ which follows from Gor'kov's derivation of the Ginzburg–Landau equations. The vector potential is measured in units $\Phi_0/2\pi\xi(0)$ ($\Phi_0 = ch/2e$ is the quantum of magnetic flux). In these units the magnetic field is scaled with $H_{c2}(0) = \Phi_0/2\pi\xi(0)^2$ and the current density with $j_0(0) = c\Phi_0/8\pi^2\lambda(0)^2\xi(0)$. We use the model as stated in Ref. [35], but our normalization is relative to the variables at $T = 0$. This results in the explicit inclusion of normalized temperature T in the first equation, as found, e.g., in Refs. [39,41]. The first equation governs the relaxation of the superconducting order parameter ψ and the second equation is the Maxwell equation for magnetic induction field $\mathbf{B} = \nabla\times\mathbf{A}$.

Following commonly used assumptions and notations (see, e.g., Ref. [37]), in the Cartesian reference frame shown in the lower left inset of Fig. 1 our superconducting slab is assumed to have a finite normalized width (or thickness) in the x direction $w \equiv W/\xi(0) > 2\kappa$ with κ quite larger than $1/\sqrt{2}$ (definitely type II superconductor) and normalized extension in the y and the z directions much greater than width w, such that the extension in the y and the z directions can be mathematically assumed as infinite. The transport current density is directed as the y axis, $\mathbf{J} = (0, J, 0)$. The applied magnetic field is directed as the z axis, $\mathbf{H} = (0, 0, H_z)$, and it is assumed to be uniform along y and z but antisymmetric with respect to the middle of the slab, i.e., $H_z(-x) = -H_z(x)$. As a simple analytical expression we take $H_z(x) = H\sin(x/R)$. An example of such an antisymmetric magnetic field distribution is given in the main panel of Fig. 1. To simplify the analysis, we assume that the maximum of the applied field strength falls out of the edges $x = \pm w/2$ of the slab. This implies that relation $R > w/\pi$ should be satisfied. In the used geometry the dependence of physical quantities on z direction can be neglected [37] and therefore the problem becomes two-dimensional, as shown in the lower right inset of Fig. 1. Hence, the general model Eqs. (1) and (2) reduces to:

$$u\frac{\partial\psi}{\partial t} = (\nabla_{2D} - i\mathbf{A})^2\psi + \left(1 - T - |\psi|^2\right)\psi, \quad (3)$$

$$\frac{\partial A_y}{\partial t} = J_{sy} + \kappa^2\frac{\partial^2 A_y}{\partial x^2} + \kappa^2\frac{\partial^2 A_y}{\partial y^2} - \kappa^2\frac{\partial H_z}{\partial x}, \quad (4)$$

where $\nabla_{2D} \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$ is the two-dimensional gradient operator and $J_{sy} = \frac{1}{2i}(\psi^*\nabla_y\psi - \psi\nabla_y\psi^*) - |\psi|^2A_y$ is the y-component of the supercurrent density $\mathbf{J}_s$. Notice that the external field described by $H_z = H\sin(x/R)$ explicitly appears both in the bulk of Eq. (4) and in the boundary conditions for the vector potential $\mathbf{A}$. To simulate a sample extension in the y direction much larger than the sample width, we apply periodic boundary conditions [37] in the y-direction, both for the vector potential and the order parameter: $A_y(x, y + \Lambda/2) = A_y(x, y - \Lambda/2)$ and $\psi(x, y + \Lambda/2) = \psi(x, y - \Lambda/2)$, where Λ is the spatial period along y-direction (see Fig. 1). The superconductor–vacuum boundary [17] conditions for the order parameter are instead applied in the x-direction. These conditions, stating that the normal component of the supercurrent is zero at the boundary Γ of the superconductor (supercurrent cannot exit from the superconductor) are mathematically expressed [17] as $(\nabla_{2D} - i\mathbf{A})\psi\cdot\mathbf{n}|_\Gamma = 0$. Here these conditions simplify in $\nabla_x\psi|_{x=\pm w/2} = 0$, meaning $J_{sx}|_{x=\pm w/2} = 0$. The bias current is introduced through the boundary condition for $\mathbf{A}$ in the z-direction: $(\nabla\times\mathbf{A})_z|_{x=\pm w/2} = H_z(x = \pm w/2) \mp H_J$, where $H_z(x = \pm w/2)$ is the z component of the applied magnetic field and $H_J = Jw/2\kappa^2$ is the z component of magnetic field generated by the bias current density J (self-field of transport current) at the edges of the slab.

We choose parameters $\kappa = 5,\ T = 0.8,\ w = 40,\ R = 1.6w$, and the spatial period along y-direction $\Lambda = 20$. To numerically solve the system of Eqs. (3) and (4) we apply a finite-difference representation for the order parameter and vector potential on a uniform Cartesian space grid with step size 0.5 and we use the link variable approach [35,36] and the simple Eulero method [42] with time step $\triangle t = 0.002$ to find ψ and $\mathbf{A}$. Initial conditions are $|\psi| = 1$ and $\mathbf{A} = 0$. The behavior of the system is studied on a large time scale when time-averaged values no longer depend on time.

Before we proceed, we would notice that the slab geometry has been often used to extract significant physics in fundamental textbooks [17,43–45] of superconductivity, in seminal works [46,47] on the subject of flux entry and flux exit, and in magneto-transport properties of superconductors in the specific framework [34–37,31,38] of Ginzburg Landau model. This is because it describes with a good approximation a very common experimental configuration: a superconducting film of finite thickness (eg., up to tenths of a micron) with lateral dimensions very large (e.g., tens or hundred of microns) with respect to its thickness and subjected to a magnetic field oriented parallel to the substrate.

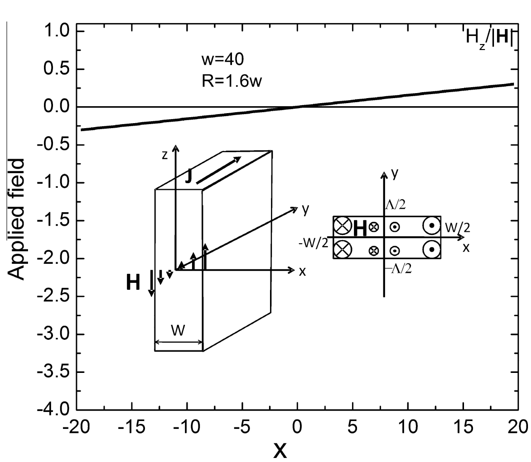

Fig. 1. In the main panel there is plotted the spatial distribution of the magnetic field applied to the superconducting slab sketched in the left inset. The extension of the slab in the y and z directions is assumed much larger than width W, so that it can be considered infinite at desired approximation. In the right inset there is shown the 2D simplified representation of the slab we have adopted.