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# The origins of macroscopic quantum coherence in high temperature superconductivity

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ABSTRACT

A new, theoretical approach to macroscopic quantum coherence and superconductivity in the *p*-type (hole doped) cuprates is proposed. The theory includes mechanisms to account for e-pair coupling in the superconducting and pseudogap phases and their inter relations observed in these materials.

Electron pair coupling in the superconducting phase is facilitated by local quantum potentials created by static dopants in a mechanism which explains experimentally observed optimal doping levels and the associated peak in critical temperature. By contrast, evidence suggests that electrons contributing to the pseudogap are predominantly coupled by fractal spin waves (fractons) induced by the fractal arrangement of dopants.

On another level, the theory offers new insights into the emergence of a macroscopic quantum potential generated by a fractal distribution of dopants. This, in turn, leads to the emergence of coherent, macroscopic spin waves and a second associated macroscopic quantum potential, possibly supported by charge order. These quantum potentials play two key roles. The first involves the transition of an expected diffusive process (normally associated with Anderson localization) in fractal networks, into e-pair coherence. The second involves the facilitation of tunnelling between localized e-pairs. These combined effects lead to the merger of the super conducting and pseudo gap phases into a single coherent condensate at optimal doping. The underlying theory relating to the diffusion to quantum transition is supported by Coherent Random Lasing, which can be explained using an analogous approach. As a final step, an experimental program is outlined to validate the theory and suggests a new approach to increase the stability of electron pair condensates at higher temperatures.



## 1. Introduction

In conventional superconductors (SC), electrons are bound through interactions between electrons and quanta of lattice vibrations (phonons), to form a bosonic fluid of electron-pairs (e-pairs), in a mechanism first described by Bardeen, Cooper, and Schrieffer (BCS) [1–3]. The relatively long range coherence length of e-pairs is influenced by material characteristics but also by an increase in the de Broglie wave length $\lambda_{deB} = \hbar/p$ as momentum decreases with decreasing temperature. The coherence length permits large numbers of e-pairs to occupy the same density of states (DOS) creating a lower entropy, phase coherent, macroscopic quantum state with long range order. However, the energy gap $E_g$ between condensed and normal states (due to low phonon coupling energies) is small, resulting in low critical temperatures $T_c$ ($\lesssim 20$ K) when compared with High Temperature Super Conducting (HTSC) materials. After more than two decades of intense experimental and theoretical research, there is still no widely accepted theory to explain macroscopic quantum coherence and the significantly higher electron coupling energies in these high temperature superconductors. In this paper we review a number of studies offering some key insights, which we consider in the development of a new theoretical approach to explain HTSC properties.

Due to the enormous number of papers published on a range of different materials with different characteristics, we have focussed on the most studied *p*-type (hole doped) family of cuprates. However, where appropriate we have indicated how we think the theory may have applicability to the broader family of HTSC materials.

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## 2. Background

### 2.1. Structural geometry and its influences

The symmetry of a structure on its microscopic scale is an important property of condensed matter systems. Fluids, glasses or amorphous solids have dispersion relations with a rotational symmetry, whilst crystalline materials are anisotropic, leading to a spatial anisotropy of DOS, with particle momentum being geometrically constrained. One of the more notable aspects of HTSC materials lies in their disordered structure with a number of papers [4–12], indicating that HTSC is favoured by complex fractal systems. A key objective of this current work is to explore a possible link between the geometry of these materials and their unusual properties.

As part of the discussion of quantum coherence in disordered materials we include the recently discovered phenomena of Coherent Random Lasing (CRL), which occurs in fractal media under very specific geometries [4]. As with HTSC, no single theory has yet been universally accepted to explain CRL. However, there are some obvious parallels between the two phenomena, with a clear link between network geometry and macroscopic quantum coherence in bosonic fluids.

Considering a Schrödinger equation describing electrons moving in a random potential, and the scalar wave equation for light propagation in a medium with fluctuating dielectric constant [4], both equations can be presented in a generic form

$$\Delta\psi(x) + [k - U(x)]\psi(x) = 0. \tag{1}$$

For an electron with mass $m$ and energy $E$, moving in a random potential $V(x)$, the parameters $k$ and $U(x)$ are $k = 2mE$, $U(x) = 2mV(x)$. For a light wave with frequency $\omega$, travelling in a medium with dielectric constant $\epsilon$, corresponding expressions for $k$ and $U(x)$ take the form $k = \epsilon(\frac{\omega}{c})$, $U(x) = -\delta\epsilon(x)(\frac{\omega}{c})^2$ where $\delta\epsilon(x)$ is the fluctuating part of the dielectric constant. On an ordered lattice with all wells the same depth, an electron is mobile for a range of energies. However, in disordered media, well depths become more random and electrons with sufficient negative energy, may get trapped in regions where the random potential $V(x)$ is particularly deep [9,13]. The ability of electrons to tunnel out of the potential well depends on the probability of finding nearby potential fluctuations into which the trapped electron can tunnel.

If we consider $V(x)$ to have a root-mean-square amplitude $V_{rms}$ and a length scale $a$ (the minimum scale of the network which relates to size of the basic elements or conducting links constituting the structure), on which random fluctuations in the potential take place, $E_a$ is the conduction band width which is defined by an energy scale $E_a \equiv \hbar^2/(2ma^2)$. At the weak disorder limit, $V_{rms} \ll E_a$, a transition takes place at a critical energy $E_c \simeq -V_{rms}2/E_a$. Successive tunnelling events allow electrons of energy $> E_c$ to traverse the entire solid in a diffusive process leading to conductivity, whilst, electrons with energy $< E_c$ are trapped and do not conduct. For energies $\gg E_c$, the electron traverses the solid with relative ease, provided disorder is below a critical level. However, unlike conventional conduction, e-pairs in SC are additionally constrained by thermal instability above $T_c$.

In the case of HTSC, below $T_c$, the quantum tunnelling length for an electron pair is given by $l \sim \hbar\sqrt{4mT}$, where $T$ stands for the height of the barrier. A superconducting state may therefore exist if $l \gtrsim a$ according to the relation

$$T \lesssim \hbar^2/4ma^2. \tag{2}$$

By contrast with electrons, a binding potential well does not exist for light. However in a second localization mechanism originally described by Anderson [14], photons (and electrons) can still be trapped in disordered networks. At the weak disorder limit, when a quantum particle is inserted in a disordered system it will start to spread in a diffusive process, the wave being backscattered by impurities leading to weak localization effects. The multiple scattering of the wave can enhance these perturbative effects to such a degree that they become spatially localized. Below a critical level of disorder, there is a finite probability for the particle to return to the point at which it was inserted. At very high levels of disorder, states become exponentially extended and this probability moves to zero. However, due to the lack of a binding potential, photons have the ability to theoretically escape from disordered media, although in practice they can be trapped for extended periods of time [4].

CRL emerges under very specific conditions as the degree of disorder increases [4]. In the absence of mirrors, which are normally required to support coherent lasing, the disordered medium itself somehow takes on this role. For the disordered medium to play the role of a Fabry–Perot resonator, it is necessary that certain Eigen functions are completely, or almost localized. An almost localized solution can be viewed as a very high local maximum of the extended Eigen function $\psi(x)$. If this maximum is viewed as a core, then the delocalized tail can be viewed as a source of leakage. The higher the local maximum of $\psi(x)$, associated with increasing levels of disorder, the longer the period of localization. At low levels of gain, leakage is not observable. However, as gain increases this changes, with phase coherent photons escaping at multiple locations, observed as random lazing.

We conclude that established theory on localization in disordered networks appears to conflict with HTSC and CRL, which are supported by high levels of disorder. A new theory is required to address this specific issue. Alpakov [4] suggested that the link to the Fabry–Perot resonator could be identified with disorder-induced resonance leading to macroscopic quantum coherence. Milovanov and Rasmussen [9] hint at a mechanism whereby the complex microscopic texture of heterogenous HTSC materials support fractional harmonic modes that could lead to macroscopic coherence. In both HTSC and CRL, a common theme of macroscopic resonance in disordered networks emerges which we consider in Section 3.

### 2.2. Fractal networks

In considering fractal structures we define three specific dimensions. The dimension of the embedding Euclidean space $E^D$ where $D$ is the integer dimension, the fractal (Hausdorff) dimension $D_F$ and the spectral (Fracton) dimension $D_{fr}$. Fractons are localized quantum oscillations of the local disorder in the system in a manner similar to extended modes (e.g. phonons) found in an ordered lattice [5,9,15,16,18,19].

When the concentration $q$ of a fractal web embedded in $E^D$ goes to zero ($q \to 0$), there are no interconnecting conducting elements and zero conduction. Conversely, as $q \to 1$, a fractal web densely fills the Euclidean space as a percolating, multiscale, infinitely connected web. At some point on the continuum from $0 \to 1$, $q$ reaches a minimum critical concentration $q_c$. At this 'percolation threshold', we have an infinite connected fractal web occupying a fraction of $E^D$, which conducts on large scales [9]. At some point $> q_c$ we expect to see the emergence of disorder induced localization effects.

### 2.3. Electron-pair coupling mechanisms

Büttner and Blumen [5] were amongst the first to suggest that HTSC could be supported by a partial (or total) fractal lattice with e-pair coupling being mediated by fractal phonon