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Fabry–Perot filters with tunable Josephson junction defects

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ABSTRACT

We propose to take advantage of the properties of long Josephson junctions to realize a frequency variable Fabry–Perot filter that operates in the range 100–500 GHz with a bandwidth below 1 GHz. In fact, we show that it is possible to exploit the tunability of the effective impedance of the Josephson component, that is controlled by a dc bias, to tune, up to 10% of the central frequency, the resonance of the system. An analysis of the linearized system indicates the range of operation and the main characteristic parameters. Numerical simulations of the full nonlinear Josephson element confirm the behavior expected from the linear approximation.

1. Introduction

Layered structures have been introduced in microwave and quasi-optical devices to produce filters with a narrow transmission bandwidth [1] and as filter in the high frequency domain [2]. Layered structures, periodic or aperiodic, have also been proposed for fundamental physics applications with sophisticated optics [3] and microwave applicators [4]. A frequency band can be selected introducing, in the terminal points of the periodic structures, suitable defects; the structures thus constitute a sort of Fabry–Perot filter in the optical and quasioptical band that alternate periodically two different sections H and L of high and low distributed capacitance, respectively. Fabry–Perot like devices provide high precision measurements set-up for demanding applications [5,6]. The defect ordinarily consists in a linear Transmission Line (TL), thus a well known (and commonly employed) system can be modeled as a periodic array of $\lambda/4$ TL that introduces identical delays, while the defect consists in still another TL of different length. The periodic structure determines a band gap; if one introduces a defect, a narrow resonance appears in the bandwidth. A natural limit of this technology is that the resonant frequency is determined by the physical characteristics of the defects; therefore, to tune the central frequency one should build (and insert) a different defect. Therefore, to overcome this complication superconducting tunable filters (with lumped parameters) have been explored to realize electronically controlled devices without mechanical tuning [7–9]. In this context, we propose to employ as defects Josephson Junctions Transmission Lines (JJTL) [10–12] to obtain a tunable filter in the sub-THz region. The motivation is twofold: Josephson Junctions (JJ) are very fast superconducting elements, suitable to build high frequency and low noise transmission lines [13–15], pseudocavities [16], and generators [17–20] capable to perform even near the THz region. Moreover, JJ's are nonlinear element whose (effective) inductance can be tuned by an external dc bias [12,21]. The latter property of JJ allows to control the effective inductance, at high frequency, with a change of the direct current through the superconducting element [22,23]. In this work we examine how the main properties of such electronically controlled defect can be exploited to design a tunable filter. The aim is to show that one can tune some properties of the device with a (relatively) simple change of the applied current. The main superconducting TL Josephson element consist of a so-called long JJ [10,11]. Design capability for JJ are highly developed, as demonstrated by the recent upsurge of superconducting metamaterials [24,25], especially for quiet quantum measurements [12,26]. It is therefore conceivable that the proposed concept can be considered for practical purposes. The work is organized as follows. In Section 2 we retrieve, in the *ABCD* matrix formalism [27], the dependence of the JJTL as a function of the dc bias. In Section 3 we show in the linear approximation the main properties of the proposed structure: bandwidth and tunability. We also numerically investigate the full nonlinear JJTL and compare its behavior to the approximated linear analysis. Section 4 concludes.

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2. Model

The constitutive equations for the JJTL element of Fig. 1 read [24]:

$$\frac{\partial V}{\partial z} = -L_0 \frac{\partial I}{\partial t} \tag{1}$$

$$\frac{\partial I}{\partial z} = -C_j \frac{\partial V}{\partial t} - I_c \sin\phi + I_b - G_j V \tag{2}$$

$$\frac{\hbar}{2e} \frac{\partial \phi}{\partial t} = V \tag{3}$$

where $\hbar$ is the Planck constant, e is the elementary charge, V and I are the distributed voltages and currents, respectively; C_j, G_j, and I_c are the capacitance, conductance and maximum (or critical) current per unit length of the superconductive element; L_0 is the geometric inductance per unit length. Eqs. (1)–(3) amount to a sine–Gordon equation [10,11]. The variable ϕ is the gauge invariant phase difference between the superconducting wave functions [10] that determines the Josephson supercurrent $I_c \sin(\phi)$, the essence of the Josephson effect, as it corresponds to the tunneling of Cooper's pairs. Finally, Eq. (3) is the phase-voltage Josephson relation. From the standpoint of the electromagnetic propagation it corresponds to a nonlinear distributed inductance [9]:

$$L_j = \frac{\hbar}{2eI_c}\left[1 - \left(\frac{I_b}{I_c}\right)^2\right]^{-1/2}. \tag{4}$$

This is the key property of the Josephson effect that we want to exploit: the effective inductance L_j can be tuned by the external dc bias I_b, in analogy to fluxometers (SQUIDs) or other microwave components [22].

Linearizing the JJTL Eqs. (1)–(3) and using the distributed inductance (4) in the frequency domain we find:

$$\frac{dV}{dz} = -j\omega L_0 I \tag{5}$$

$$\frac{dI}{dz} = -\left(j\omega C_j + \frac{1}{j\omega L_j} + G_j\right)V \equiv -j\omega C_{eq} V, \tag{6}$$

where V and I are the voltage and current phasors and we use the substitution rule $\partial(\cdot)/\partial t \rightarrow j\omega$. The factor C_{eq} in Eq. (6) amounts to an equivalent condenser $C_{eq} \equiv C_j - 1/(\omega^2 L_j) + G_j/j\omega$, that depends on the dc bias I_b through Eq. (4). Through the whole paper we use the term *frequency* as a synonymous of *angular velocity*. The linear analysis of the TL gives the (secondary) parameters β and Z_c:

$$\beta = \omega\sqrt{L_0 C_{eq}}, \tag{7}$$

$$Z_c = \sqrt{\frac{L_0}{C_{eq}}}. \tag{8}$$

In the *ABCD* matrix formalism [27] the parameters β and Z_c are fundamental to derive the input–output matrix relations. For instance, the parameters β and Z_c of Eqs. (7) and (8) can be employed to derive the input–output relations of a JJTL:

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} \cos(\beta l) & jZ_c \sin(\beta l) \\ jZ_c^{-1}\sin(\beta l) & \cos(\beta l) \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix}. \tag{9}$$

In the following we shall indicate the matrix of the above Eq. (9) with $\hat{D}$.

The defect described by Eq. (9) has the structure of an ordinary defect [1] that introduces an appropriated phase delay of the signal between the input and the output points [30]. This defect can be combined with standard input–output relations for $\lambda/4$ elements [27]. If L and H are two transmission lines with different capacitances (C_L and C_H for the low and high pieces, respectively) and with the same inductance L_{ind}, one obtains a band pass behavior due to the periodic structure. For the complete system (including the JJTL defect) of Fig. 1, the transmission matrix reads:

$$\begin{aligned} T &= \underbrace{LHLH\ldots LHLH}_{\mathrm{N_L\ times}}\ \hat{D}\ \underbrace{LHLH\ldots LHLH}_{\mathrm{N_L\ times}}\ \hat{D}\ \hat{R}_L \\ &= (LH)^{N_L}\hat{D}(LH)^{N_L}\hat{D}\ \hat{R}_L \end{aligned} \tag{10}$$

that completely describes the linear behavior (here, a termination is described by the resistive matrix $\hat{R}_L$). If we rewrite T, i.e. the result of the matrix multiplication in Eq. (10) using Eq. (9), as

$$T = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}, \tag{11}$$

the input impedance Z_i reads:

$$Z_i = \frac{V_i}{I_i} = \frac{A'R_L + B'}{C'R_L + D'}. \tag{12}$$

Finally, denoting with R_G the impedance (purely resistive) of the feeding TL, the transmission coefficient Γ_i reads:

$$\Gamma_i = \frac{Z_i - R_G}{Z_i + R_G}. \tag{13}$$

Eq. (13), together with Eq. (4) that is contained in the matrix $\hat{D}$, states that the transmission coefficient exhibits a dip that depends upon the dc bias current I_b. This is the main result of this paper, illustrated in the next Section.

3. Results

To characterize the proposed Fabry–Perot filter, we include an input impedance and a load to obtain relevant quantities as the transmission coefficient. For simplicity, we consider a simple voltage source and a resistive load at the beginning and the end of the block diagram described in Section 2. For the parameters of Table 1 the reflection coefficient $|\Gamma_i|$ as a function of the frequency is displayed in Fig. 2a at two values of the bias. We stress that the resonance dip in Fig. 2a can be tuned by a simple change of the bias current $\gamma = I_b/I_c$ in Eq. (2), as shown in Fig. 2b. It appears that the central frequency can be tuned, within the allowed range of

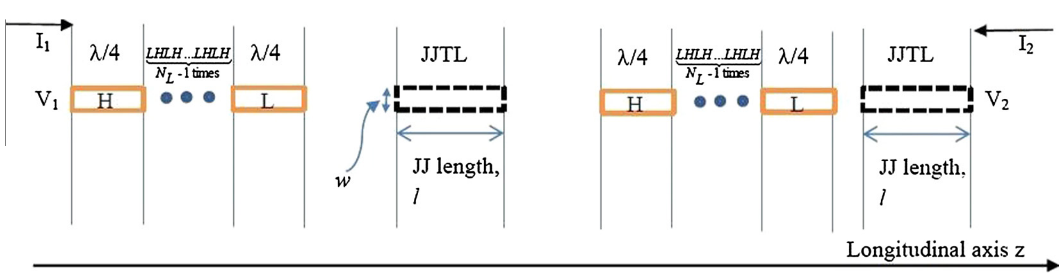

Fig. 1. Schematic top view (not in scale) of the planar microstrip circuit representing a chain of $\lambda/4$ standard TL (solid lines rectangles) joined to JJTL defects (dashed lines rectangles). The ac component travels along the z-axis, for the space between the sections is only shown to distinguish the various elements. A dc current bias is only fed through the JJ element. In the following of the paper we take the number of TL sections $N_L = 8$.