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Transport ac loss in a rectangular thin strip with power-law $E(J)$ relation

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ABSTRACT

Transport ac losses of a rectangular thin strip obeying relation $E/E_c = (J/J_c)^n$ with a fixed critical current I_c and $n = 5, 10, 20, 30$, and 40 are accurately computed at a fixed frequency f as functions of the current amplitude I_m. The results may be interpolated and scaled to those at any values of I_c, f, and $5 \leqslant n \leqslant 40$. Normalized in the same way as that in Norris' analytical formula derived from the critical-state model and converting f to a critical frequency f_c, the modeling results may be better compared with the Norris formula and experimental data. A complete set of calculated modeling data are given with necessary formulas to be easily used by experimentalists in any particular case.

1. Introduction

Soon after Bean assumed in the critical-state (CS) model a constant critical current density J_c to calculate the magnetization curve of a hard superconducting cylinder [1], London reported his derivation on the transport ac loss of a hard superconducting cylinder based on the same assumption [2]. For a long cylinder of radius a carrying a transport current $I(t) = I_m \sin 2\pi ft$, London obtained a relation between the ac loss Q per cycle per unit length and I_m as

$$q = (2 - i_m)i_m + 2(1 - i_m)\ln(1 - i_m), \quad (1)$$

where

$$q \equiv 2\pi Q/\mu_0 I_c^2, \quad i_m \equiv I_m/I_c, \quad (2)$$

$I_c = \pi a^2 J_c$ being the critical current. The validity of this formula was later extended by Norris to an elliptical bar with any values of semi-axes a and b [3]. In recent studies on 1G high-temperature superconducting (HTS) tapes prepared by the powder-in-silver tube technique, Eq. (1) for thin elliptical tapes has been well verified by both numerical calculations and analytical derivation [4,5]. Norris also derived a formula for a rectangular thin strip [3], and with the same normalization as in Eq. (2) it is expressed by

$$q = 2[(1 + i_m)\ln(1 + i_m) + (1 - i_m)\ln(1 - i_m) - i_m^2]. \quad (3)$$

This equation is more relevant to the 2G HTS tapes, for which a HTS film is epitaxially deposited on a metallic substrate with a number of buffer layers in between.

It has been shown that the measured q vs i_m curves of HTS tapes are roughly located around the modeling curves calculated from Eqs. (1)–(3) [5–15], which means that the CS model is basically valid for such tapes. However, this comparison between experimental and modeling results is somewhat ambiguous, since the relations between current density J and electrical field E in the actual HTS tapes and in the CS model are significantly different. In the CS model, $E = 0$ occurs when $|J| < J_c$ and a finite E appears when $|J| = J_c$. As a result, I_c for a tape is a fixed value independent of the voltage along the tape and the ac loss Q is f independent. On the other hand, in transport current–voltage (I–V) measurements of most HTS tapes, I changes with V in the full penetration regime following roughly a power law (PL), $V \propto I^n$, so that I_c has to be defined as that when $E = V/l = E_c$, where l is the distance between both voltage taps and criterion $E_c = 10^{-4}$ V/m is routinely defined. Related to this, ac loss Q is intrinsically f dependent.

The PL I–V curve comes from a PL $E(J)$ relation, which is a characteristic of collective flux creep and expressed as [16–19]

$$\mathbf{E} = (E_c/J_c)\,|J/J_c|^{n-1}\mathbf{J}. \quad (4)$$

The ac loss Q and I–V curve of a cylinder obeying PL Eq. (4) have been calculated by Chen and Gu [20,21], using a numerical technique proposed by Brandt and developed for the transport case by Rhyner and Yazawa et al. [22,23]. A scaling law has been derived in terms of vector potential, so that for any given value of n, the $q(i_m)$ function calculated at fixed I_c and f may be used for any

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values of I_c and f. In particular, the PL $q(i_m)$ is scaled to the CS $q(i_m)$ at a point $q = i_m = 1$ by converting f to a critical frequency f_c for any value of n between 5 and 30, so that the experimental results may be unambiguously compared with both the CS and PL models. We emphasize that f_c to be called the critical frequency is not because this f_c is a characteristic of CS curve itself, which is f independent, but because the highly f dependent PL curve is converted to a unique one for each value of n that has a common characteristic point with the *critical-state* curve.

We will calculate the ac loss for a rectangular thin strip in the present work and show numerically the validity of the above-mentioned scaling law. Together with the results for cylinders, the calculated results will be compared with experimental data of HTS tapes.

In Appendix A, the derivations and discussions on the scaling law presented in [20] for a cylinder will be improved with important corrections. This is necessary since it is still difficult for us to completely derive the same scaling law for a thin strip. Since the scaling law for the magnetic case is sample shape independent, as stated by Brandt in [18], we believe that the scaling law for the transport case derived for a cylinder should also be valid for a thin strip.

2. Computation of ac loss

2.1. Computation procedure

The studied rectangular thin strip is placed along the z axis with an arbitrarily fixed width $w = 10$ mm along the x axis and a very small thickness $d \ll w$. It is characterized by a PL $E(J)$ relation as Eq. (4). We calculate Q at different values of n, I_m/I_c, and f by applying the procedure described in [23]. Defining the line density of the current $K = Jd$, Eq. (4) is written

$$\mathbf{E} = (E_c/K_c)\,|\,K/K_c|^{n-1}\mathbf{K}, \tag{5}$$

where K_c is the critical current line density. The critical current and frequency are fixed as $I_c = J_c wd = K_c w = 60$ A and $f = 5$ and 50 Hz. Dividing the width w into N equal elements, each centered at $x_i (i = 1, 2, \ldots, N)$, the computation is started by calculating a matrix of components [19]

$$\begin{aligned} Q_{ij} &= \ln|x_i - x_j|/2\pi \quad (j \neq i) \\ &= \ln(w/2\pi N)/2\pi \quad (j = i). \end{aligned} \tag{6}$$

This matrix is defined for converting the current line density $K_j (j = 1, 2, \ldots, N)$ to vector potential $A_i (i = 1, 2, \ldots, N)$ by

$$A_i = -\frac{\mu_0 w}{N}\sum_{j=1}^{N} Q_{ij} K_j, \tag{7}$$

and the components of its reciprocal Q_{ij}^{-1} are used for calculating from K_i at time $k\Delta t (k = 0, 1, 2, \ldots)$, K_i^k, to that at time $(k+1)\Delta t$, K_i^{k+1}, by

$$K_i^{k+1} = K_i^k + \frac{N\Delta t}{\mu_0 w}\sum_{j=1}^{N} Q_{ij}^{-1}\left[K_j^k \rho(K_j^k) - E_e^k\right]. \tag{8}$$

The initial condition is set as $K_i^0 = 0 (i = 1, 2, \ldots, N)$. In Eq. (8), nonlinear resistivity is derived from Eq. (5) as

$$\rho(K_j^k) = (E_c/K_c)\,|\,K_j^k/K_c|^{n-1}, \tag{9}$$

and E_e^k is the external electrical field and is determined at each moment $(k+1)\Delta t$ to satisfy the following total current condition with accuracy of $0.001 \times I_m$:

$$\sum_{i=1}^{N} K_i^{k+1} w/N = I_m \sin 2\pi f[(k+1)\Delta t]. \tag{10}$$

Table 1
q of a rectangular thin strip of width $w = 10$ mm and critical current $I_c = 60$ A as a function of i_m and n, numerically calculated at $f = 5$ Hz.

i_m	$n = 5$	10	20	30	40
0.1	0.000270	0.0001138	0.0000681	0.0000549	0.0000487
0.2	0.00281	0.00144	0.000953	0.000813	0.000742
0.3	0.01121	0.00647	0.00458	0.00397	0.00373
0.4	0.0304	0.01910	0.01414	0.01254	0.01175
0.5	0.0671	0.0450	0.0346	0.0310	0.0292
0.6	0.1300	0.0928	0.0736	0.0668	0.0633
0.7	0.231	0.1750	0.1433	0.1314	0.1250
0.8	0.385	0.312	0.264	0.245	0.234
0.85	0.488	0.412	0.356	0.332	0.318
0.9	0.612	0.541	0.480	0.451	0.444
0.95	0.760	0.708	0.654	0.624	0.604
1	0.937	0.925	0.906	0.893	0.884
1.05	1.150	1.232	1.437	1.747	2.22
1.1	1.405	1.699	2.77	5.23	
1.2	2.09	3.52			
1.3	3.09	7.66			

The loss per meter length at each time step $k\Delta t$, Q_k, is calculated by

$$Q_k = \sum_{i=1}^{N} (K_i^k)^2 \rho(K_i^k)\frac{w\Delta t}{N}. \tag{11}$$

Many time steps are continuously calculated, and the final loss per cycle per meter length is calculated by

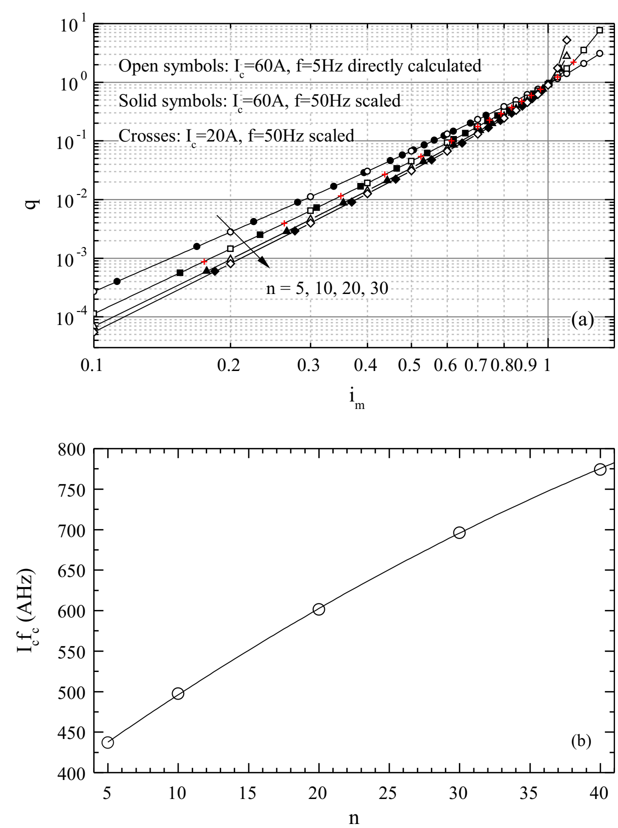

Fig. 1. (a) The $q(i_m)$ functions directly calculated at $I_c = 60$ A and $f = 5$ Hz expressed by open symbols with connected lines, compared with those directly calculated at $I_c = 60$ A and $f = 50$ Hz and then scaled to $I_c = 60$ A and $f = 5$ Hz with $C = 0.1$, expressed by solid symbols, and with those directly calculated at $I_c = 20$ A and $f = 50$ Hz and then scaled to $I_c = 60$ A and $f = 5$ Hz with $C = 0.3$, expressed by crosses. Arrow indicates the direction of increasing n. (b) $I_c f_c$ as a function of n. Symbols are obtained from the comparison of the scaled data at $n = 5, 10, 20, 30$ and 40 with Norris Eq. (3), and their fitting curve is expressed by Eq. (14).