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Proposed experimental test of an alternative electrodynamic theory of superconductors

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ABSTRACT

An alternative form of London's electrodynamic theory of superconductors predicts that the electrostatic screening length is the same as the magnetic penetration depth. We argue that experiments performed to date do not rule out this alternative formulation and propose an experiment to test it. Experimental evidence in its favor would have fundamental implications for the understanding of superconductivity.

It is not generally recognized that the conventional London electrodynamic description of superconductors [1] involves *two independent assumptions*, and that an alternative plausible formulation exists that is consistent with the Meissner effect but unlike the conventional formulation allows for the presence of electric fields in the interior of superconductors [2,3]. Here we argue that this alternative formulation has not been subject to experimental test, discuss why this an important question to settle, and propose an experiment to do so.

The conventional derivation of London's electrodynamic equations for superconductors starts from the "acceleration equation"

$$\frac{\partial \vec{v}_s}{\partial t} = \frac{e}{m_e}\vec{E} \quad (1)$$

with $\vec{v}_s$ the superfluid velocity, $\vec{E}$ the electric field and e and m_e the superfluid carriers' charge and mass. The electric current $\vec{j}_s = en_s\vec{v}_s$, with n_s the density of superfluid carriers, then obeys

$$\frac{\partial \vec{j}_s}{\partial t} = \frac{n_s e^2}{m_e}\vec{E} = \frac{c^2}{4\pi\lambda_L^2}\vec{E} \quad (2)$$

with $\lambda_L = (m_e c^2/4\pi n_s e^2)^{1/2}$ the London penetration depth. Taking the curl on both sides, using Faraday's law, integrating in time and setting the integration constant equal to zero yields the London equation

$$\vec{\nabla} \times \vec{j}_s = -\frac{c}{4\pi\lambda_L^2}\vec{B} \quad (3)$$

which, when combined with Maxwell's equation $\vec{\nabla} \times \vec{B} = (4\pi/c)\vec{j}_s$ yields

$$\nabla^2\vec{B} = \frac{1}{\lambda_L^2}\vec{B} \quad (4)$$

and hence predicts that magnetic fields can only penetrate a superconductor up to a distance λ_L from the surface.

Integration of Eq. (3) yields

$$\vec{j}_s = -\frac{c}{4\pi\lambda_L^2}\vec{A} \quad (5)$$

where $\vec{A}$ is the magnetic vector potential. Taking the time derivative on both sides of Eq. (5) and using Faraday's law yields

$$\frac{\partial \vec{j}_s}{\partial t} = \frac{c^2}{4\pi\lambda_L^2}(\vec{E} + \vec{\nabla}\phi) \quad (6)$$

where ϕ is the electric potential. Eq. (6) differs from Eq. (2) in that it allows for the presence of an electrostatic field in the interior of a superconductor, which, since $\vec{E} = -\vec{\nabla}\phi$ in a time-independent situation, will not give rise to an infinite current as Eq. (2) predicts. London and London [2] pointed out that Eq. (3) has a greater degree of generality than Eq. (2) does, in other words that Eq. (2) can be derived from Eq. (3) only under the *additional independent assumption* that $\vec{\nabla}\phi = 0$ in the interior of the superconductor, or equivalently that no electrostatic fields exist inside the superconductor. Note also that Eq. (1), from which Eq. (2) was derived, does not follow from Newton's equation, rather Newton's equation yields Eq. (1) with the *total* time derivative rather than the partial time derivative on the left side. As a consequence, Eq. (6) *is* compatible with Newton's equation [3].

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The conventional formulation of London electrodynamics [1] assumes $\vec{\nabla}\cdot\vec{A}=0$ in Eq. (5), which implies that no electric field nor charges can exist inside superconductors. The alternative formulation assumes that $\vec{A}$ in Eq. (5) obeys the Lorenz gauge and leads to the following equations for the charge density and electrostatic field in the interior of superconductors:

$$\nabla^2(\rho-\rho_0)=\frac{1}{\lambda_L^2}(\rho-\rho_0) \tag{7a}$$

$$\nabla^2(\vec{E}-\vec{E}_0)=\frac{1}{\lambda_L^2}(\vec{E}-\vec{E}_0) \tag{7b}$$

with either $\rho_0=0, \vec{E}_0=0$ [2,4–8], or ρ_0 a positive constant, with $\vec{\nabla}\cdot\vec{E}_0=4\pi\rho_0$ [3]. $\rho_0>0$ implies that the charge distribution in superconductors is macroscopically inhomogeneus, with excess negative charge near the surface and a radial electric field in the interior [3].

Eq. (7) with either $\rho_0=0$ or $\rho_0\neq 0$ implies that the screening length for applied electrostatic fields in superconductors is λ_L, typically several hundreds Å, much longer than the Thomas Fermi screening length in normal metals, typically of order Å. H. London attempted to test the validity of Eq. (7) experimentally [9] by looking for changes in the capacitance of a capacitor where the mercury electrodes changed from normal to superconducting as the temperature is lowered. He hypothesized that if the electric field penetrates a distance $\delta\sim\lambda_L$ into each electrode, the effective distance between electrodes would be increased by $\sim 2\lambda_L$, leading to a measurable decrease in the capacitance. He detected no change, and based on this result the London brothers concluded [9,10] that the electric field does not penetrate a superconductor, hence that conventional London electrodynamics, with $\vec{\nabla}\cdot\vec{A}=0$, describes superconductors in nature.

In this paper we argue that H. London's test could not have detected whether Eq. (7) is valid. Furthermore we argue than no subsequent experiment has tested Eq. (7). Finally we propose an experiment that can rule out or confirm Eq. (7).

Consider a superconducting electrode in a capacitor subject to a uniform electric field E normal to its surface as shown in Fig. 1. We argue that the negatively charged superfluid as a whole will rigidly shift with respect to the positive ionic background creating a surface charge density σ that will prevent the electric field from penetrating the interior. Using $E=4\pi\sigma, \sigma=en_s\delta$ we find

$$\delta=\frac{E}{4\pi en_s}=\frac{eE}{m_ec^2}\lambda_L^2 \tag{8}$$

so e.g. for an applied electric field of 10^5 V/cm and a typical London penetration depth $\lambda_L=500$ Å the displacement required to screen the electric field is a tiny $\delta=4.5\times10^{-4}$ Å. Therefore, the electric field will not penetrate the superconducting electrode and no change in the capacitance will be detected when the electrode goes from the normal to the superconducting state. Thus, the null result of H. London's experiment is explained independent of the validity or invalidity of Eq. (7). Similarly, the null results of two recent experiments designed to test Eq. (7) [11,12] are explained by Fig. 1.

It has been argued [13] that experiments with single electron transistor devices [14] (SET's) performed in recent years [15–17] should have detected the unconventional behavior predicted by Eq. (7) if it existed. A SET consists of a small metallic island connected to leads through small-capacitance tunnel junctions, and these experiments have been performed with superconducting *Al* at temperatures well below the transition temperature [15–17]. The charging energy of the island is inversely proportional to the sum of the capacitances of the tunnel junctions involved, and would undergo an appreciable change if electric fields were to penetrate a London penetration depth when the system is cooled, and such changes have not been reported in the literature [13]. However, we argue that the geometry of these devices [18] is such that the electric fields are uniform over distances much larger than the London penetration depth, hence it allows for a rigid shift of the superfluid to screen the electric fields as shown in Fig. 1, and consequently these experiments have nothing to say about the validity or invalidity of Eq. (7).

Similarly, it has been argued [19] that experiments with superconducting microwave resonators performed in recent years [20–22] prove the invalidity of Eq. (7). The resonance frequency of these devices is inversely proportional to the square root of the capacitance of the system and should show different behavior at low temperatures than what is seen experimentally if electric fields penetrate the superconducting components a distance λ_L [19]. However, again we argue that because for these devices the electric field applied to the superconducting components is uniform over distances much larger than the London penetration depth, a rigid shift of the superfluid as shown in Fig. 1 will prevent the electric field from penetrating the superconducting components, thus not testing the validity or invalidity of Eq. (7).

To test the validity of Eq. (7) it is necessary to apply an electric field that varies over distances smaller than the London penetration depth. Consider the situation depicted in Fig. 2. A positive test charge q is placed at a distance d above a metallic film of thickness t. When the metal is in the normal state, a non-uniform surface charge density is induced on the upper surface, given by

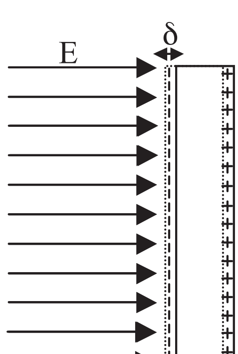

Fig. 1. Capacitor plate made of a superconducting material (solid rectangle). When a uniform electric field pointing towards the plate is applied, the negative superfluid will shift rigidly a distance δ to the left to nullify the electric field in the interior. The dotted line denotes schematically the boundary of the superfluid.

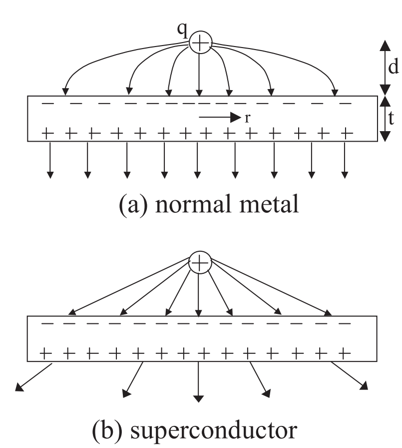

Fig. 2. Charge q at distance d from a (a) normal metal and (b) a superconducting film. The lines with arrows are electric field lines. The electric field lines in the interior of the superconductor are not shown. The electric field is zero in the interior of the normal metal.