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Fracture analysis of a transversely isotropic high temperature superconductor strip based on real fundamental solutions

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ABSTRACT

Real fundamental solution for fracture problem of transversely isotropic high temperature superconductor (HTS) strip is obtained. The superconductor *E–J* constitutive law is characterized by the Bean model where the critical current density is independent of the flux density. Fracture analysis is performed by the methods of singular integral equations which are solved numerically by Gauss–Lobatto–Chybeshev (GSL) collocation method. To guarantee a satisfactory accuracy, the convergence behavior of the kernel function is investigated. Numerical results of fracture parameters are obtained and the effects of the geometric characteristics, applied magnetic field and critical current density on the stress intensity factors (SIF) are discussed.

1. Introduction

It has been reported that cracking could occur in a large single-grain bulk superconductor when the applied field decrease from 10 to 0 T at 50 K [1]. When the cracked superconductor is subjected to a large electromagnetic force, the high stress concentration may initiate growth of the crack and may eventually lead to fracture [2]. Therefore, analysis of the crack problem is one of the most important subjects of the superconductor mechanics. There are some existing literatures on various fracture problems of the homogeneity HTS. Zhou et al. firstly studied the crack problem of a long rectangular superconductor slab under an electromagnetic force [3]. Moreover, Gao et al. studied the crack-inclusion problem for a long rectangular superconductor slab and two collinear crack problems [4,5]. Yong et al. investigated the crack problem of thin superconducting strip in a perpendicular magnetic field [6]. Recently, Gao et al. studied the dynamic fracture problem of the HTS under an alternating magnetic field [7]. All of these fracture analyses are based on the assumption that the HTS is an isotropic homogeneity material. However, superconductor is, in fact, a non-homogeneity material with different material properties in *a*, *b* and *c* directions, such as Young's modulus, thermal expansion coefficient, and thermal conductivity [8–9]. Therefore, the fracture analysis of the non-homogeneity HTS is more important than that of the assumed homogeneous HTS. In this paper, we assume the non-homogeneity HTS as a transversely isotropic material. It is worthy to introduce three basic fracture modes of the material, namely mode-I, mode-II and mode-III. Mode I is an opening mode. In the opening mode, crack surfaces are pulled apart in the normal direction. Mode-II is in-plane shear failure mode. The shear stresses act parallel to the plane of the crack. Mode-III is out of plane tearing mode. The shear stresses are applied parallel to the plane of the crack and crack front. The mode-I crack problems are generally solved analytically in the complex domain because of the complex fundamental solutions of in-plane governing equations. Actually, the mode-I crack problems can be analyzed in real domain. Li et al. in Refs. [10–12] have studied firstly the crack problem using the real fundamental solutions. The thermal fracture problem of the functionally graded coating-substrate structure has also been studied by the real fundamental solutions [13]. In this paper, the mode-I crack problem of transversely isotropic HTS is solved by a new real fundamental solutions method.

In the present work, a major effort is made to examine the effect of the electromagnetic force arising from flux pinning on the fracture behavior based on the assumption which the demagnetization effects could be neglected and that the magnetic behavior can be

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described by the critical state model [14]. The crack problem of the transversely isotropic HTS is then analyzed in real number field by recasting the solutions in real form. The fracture problem is analyzed and solved by the Cauchy singular integral equation and Gauss–Lobatto–Chybeshev collocation method.

2. Problem formulation

The problem under consideration is shown in Fig. 1. The transversely isotropic HTS strip contains a crack placed in a magnetic field. The crack of length $2a$ parallel to the upper and lower boundaries and the distance between the crack and boundaries of the strip is h. A rectangular coordinate system is established with the rightward x axis along the crack line and the upward y axis through crack center.

The electromagnetic force arising from flux pinning is body forces [14]. For a crack problem under body force loading, it is generally quite difficult to determine the stress intensity factor and to obtain the effect of the electromagnetic force on the fracture behavior. Therefore, a uniform stress σ_b is chosen as the equivalent force acting on the x–y plane [3]. The equivalent force σ_b can be described by the critical state Bean model, i.e. the critical current density J_c is set to be a constant independent of magnetic field [15]

$$\sigma_b = -\frac{1}{2h\mu_0}\int_0^h \left[(B_a)^2 - B(y)^2\right]dy = -\frac{1}{2\mu_0}\left[(B_a)^2 - B(0)^2\right] \tag{1}$$

where μ_0 is permeability, B_a and $B(0)$ are the magnetic field at $y = h$ and $y = 0$, respectively.

For a crack problem in the transversely isotropic superconductor material under electromagnetic force, the constitutive equations under plane strain have the following forms [16]

$$\sigma_{xx} = c_{11}\frac{\partial u}{\partial x} + c_{13}\frac{\partial w}{\partial y} \tag{2}$$

$$\sigma_{yy} = c_{13}\frac{\partial u}{\partial x} + c_{33}\frac{\partial w}{\partial y} \tag{3}$$

$$\tau_{yx} = c_{44}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial y}\right) \tag{4}$$

where σ and τ denote the normal and shear stress, respectively. c is the elastic constant. u and w are mechanical displacements in the x and y directions, respectively.

The boundary conditions along the upper and lower boundaries are

$$\sigma_y(x, \pm h) = 0 \tag{5}$$

$$\tau_{yx}(x, \pm h) = 0 \tag{6}$$

The boundary conditions along the crack surface of the strip are

$$\sigma_y(x, 0) = -\sigma_b \tag{7}$$

The governing equations of the transversely isotropic material can be expressed as

$$c_{11}\frac{\partial^2 u}{\partial x^2} + c_{44}\frac{\partial^2 u}{\partial y^2} + (c_{13} + c_{44})\frac{\partial^2 w}{\partial x \partial y} = 0 \tag{8-1}$$

$$c_{33}\frac{\partial^2 w}{\partial y^2} + c_{44}\frac{\partial^2 w}{\partial x^2} + (c_{13} + c_{44})\frac{\partial^2 u}{\partial x \partial y} = 0 \tag{8-2}$$

$$e_{15}\frac{\partial^2 w}{\partial x^2} + e_{33}\frac{\partial^2 w}{\partial y^2} + (e_{15} + e_{31})\frac{\partial^2 u}{\partial x \partial y} = 0 \tag{8-3}$$

Introducing the Fourier integral transform to Eq. (8) obtains the ordinary differential equations

$$c_{11}\xi^2\hat{u}(\xi, y) - c_{44}\frac{d^2\hat{u}(\xi, y)}{dy^2} + \xi(c_{13} + c_{44})\frac{d\hat{w}(\xi, y)}{dy} = 0 \tag{9-1}$$

$$\xi(c_{13} + c_{44})\frac{d\hat{u}(\xi, y)}{dy} + c_{33}\frac{d^2\hat{w}(\xi, y)}{dy^2} - c_{44}\xi^2\hat{w}(\xi, y) = 0 \tag{9-2}$$

Assume that $\hat{u}(\xi, y) = \varphi e^{\lambda\xi y}$ and $\hat{w}(\xi, y) = e^{\lambda\xi y}$ are the fundamental solutions of Eq. (9). Substituting $\hat{u}(\xi, y) = \varphi e^{\lambda\xi y}$ and $\hat{w}(\xi, y) = e^{\lambda\xi y}$ into the Eq. (9), one can obtain the characteristic equations:

$$\begin{vmatrix} (c_{11} - c_{44}\lambda^2) & (c_{13} + c_{44})\lambda \\ (c_{13} + c_{44})\lambda & c_{33}\lambda^2 - c_{44} \end{vmatrix} = 0 \tag{10}$$

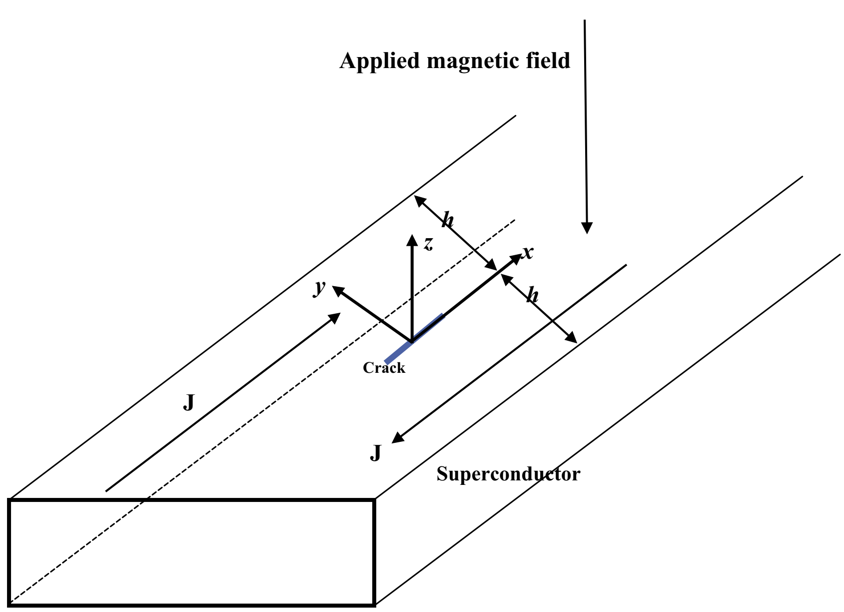

Fig. 1. Schematic of a long rectangle superconductor under magnetic field.