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Final state of thermal evolution of Jupiter-type planet

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ABSTRACT

Inside the planet consisting mostly of hydrogen (the Jupiter-type planet), in the last stage of the thermal evolution there should be induced the high-temperature superconducting state. In particular, for the effective temperature of the planet (T_S equal to about 5 K), the superconducting phase of hydrogen will appear near the value of the pressures $p_1 \equiv 500$ GPa and $p_2 \equiv 2000$ GPa. Together with the further lowering of the temperature, the superconducting state will be induced in the hydrogen layers with the pressure successively distant from p_1 and p_2. The last superconducting layer will be created for $p \simeq 400$ GPa, at the moment, when T_S drops to about 1.3 K. It has been also shown that the rotating planet, in which the superconducting state was induced, is the source of the very weak magnetic field of the induction equal to about 10^{-11} Gs. The results have been obtained on the basis of the analysis of the thermodynamic properties of the superconducting state in hydrogen subjected to the influence of the extremely high pressure at 3500 GPa (close to the core of the hypothetical planet) and on the basis of the literature describing the superconducting phase in hydrogen at the lower pressures.

1. Introduction

In all of the experimentally verified cases, the electronic superconducting state is created in rather low temperatures.

Currently, the highest critical temperature (T_C), equal to 164 K, belongs to the compound $HgBa_2Ca_2Cu_3O_{8+\delta}$, which was subjected to the influence of the external pressure (p) of the value at 31 GPa [1]. The other chemical compounds belonging to the family of the *cuprates* are characterized by the significantly lower T_C. Nevertheless, some of them have the critical temperature that is higher than the boiling point of the liquid nitrogen (e.g. the maximum critical temperature for $YBa_2Cu_3O_{7-y}$ is equal to 93.78 K [2,3]).

At this moment, the family of the hydrogenated compounds, exposed to the influence of the high pressure is also worth mentioning [4–10]. In the considered case, the theoretical predictions suggest very high values of T_C. In particular, for CaH_6 under the pressure at 150 GPa, the maximum value of the critical temperature should be equal to 243 K [11,12]. However, this result has not been experimentally verified yet.

Particularly noteworthy is the case of the pure hydrogen, in which the theoretical calculations suggest the existence of the superconducting state of the maximally high value of the critical temperature among the systems with the electron–phonon pairing mechanism. The fact above is related to the minimum mass of the nucleus of the hydrogen atom, which is equal to the mass of the single proton (m_p), and that means the abnormally high value of the Debye frequency: $\omega_D \sim \frac{1}{\sqrt{m_p}}$. Not without significance is also the lack of the internal electron shells, which contributes to the growth of the electron–phonon interaction constant.

Historically, the first publication on the subject of metallic hydrogen was the work written by Wigner and Huntington in 1935 [13]. The authors predicted that the molecular phase of hydrogen (the insulator) changes to the metallic state under the influence of the high pressure. The recent estimations have suggested that the metallization pressure for hydrogen equals about 400 GPa [14,15].

The possibility of the superconducting state induction in metallic hydrogen for the first time drew the attention of Ashcroft in 1968 [16]. Over the next years physicists dealing with the theoretical side of the issue in question have confirmed the validity of Ashcroft's hypothesis. In particular, the *ab initio* calculations suggest that the superconducting state in the molecular hydrogen would appear for $p \simeq 400$ GPa [17,18]. Most likely, it will be the highly anisotropic state with the critical temperature equal to about 80 K [19,20].

With further growth of the pressure, for the value of p equal to ~500 GPa, the dissociation of hydrogen molecular phase into the atomic phase will take place [21].

As it has been shown by the advanced numerical calculations, slightly above the pressure of the dissociation ($p = 539$ GPa), the value of the critical temperature is equal to 360 K, and that is also

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associated with the weak depairing electronic correlations, modeled by the Coulomb pseudopotential ($\mu^{\star}$) [22].

In the pressure range from 550 GPa to 800 GPa, the value of T_C of the order of 330 K should be obtained [21].

For even higher pressures, the estimation of the critical temperature was done by McMahon and Ceperley [23]. In particular, the authors found that in the pressure range from 1000 GPa to 3500 GPa, the critical temperature should increase from about 360 K to 425 K. However, two important facts should be noted: (i) the calculations conducted in the work [23] have been based on the assumption of the weak depairing electronic interactions ($\mu^{\star} \simeq 0.1$) and (ii) they do not provide the results for the pressure range from 1500 GPa to 2500 GPa (erratum to work [23]).

Referring to the discussed results, the publication of Maksimov and Savrasov [24] and the complementary work [25] are worth mentioning at this point. In particular, the results obtained in the studies [24,25] show that for $p = 2000$ GPa, the critical temperature reaches a very high value: $T_C(\mu^{\star}) \in \langle 631, 413\rangle$ K, whereas $\mu^{\star} \in \langle 0.1, 0.5\rangle$.

The properties of the superconducting state in hydrogen discussed so far were related to the critical temperature. We cannot, however, forget about the other thermodynamic quantities, such as: the order parameter, which is directly linked with the existence of the energy gap at the Fermi surface, the specific heat of the superconducting state, and the thermodynamic critical field.

With regard to the problem raised above, the results contained in the works [18,20,22,25] explicitly prove that the values of the discussed thermodynamic quantities very significantly deviate from the predictions of the classical BCS theory [26]. From the physical point of view this fact results from the existence of the strong-coupling and the retardation effects in metallic hydrogen.

The unusual properties of the hydrogen superconducting phase (most of all: a very high value of T_C) raise the question, whether the discussed state could be obtained in the laboratory. Unfortunately, most likely it will not be possible. It is not difficult to understand that the biggest problem is related to the technical difficulties in obtaining the extremely high pressures.

On the other hand, the very nature provides us with the objects, in which the discussed pressure range ($p \in \langle 400, 3500\rangle$ GPa) is achievable. We are here referring to the giant planets primarily composed of hydrogen, like Jupiter.

Note that the studies on the internal structure of Jupiter has been underway for many years [27]. Currently, it is believed that Jupiter consists mainly of hydrogen (~75%) and in about 20% of helium [28]. The admixtures of the heavier elements are concentrated most likely in the small core of the planet [27,29,30].

The effective temperature of the surface layer of Jupiter is equal to ~ 170 K, and the pressure has the value at 100 kPa. The surface layer is formed mainly by hydrogen and helium in the gas state, with the trace amounts of methane, water and ammonia. Below the pressure at 200 GPa extends the layer of molecular hydrogen, which ends with the relatively thin "inhomogeneous" layer at a temperature of ~6500 K. The deeper layer is formed by metallic hydrogen. It borders with the core of the planet, which is heated to the temperature at ~21,000 K, with the pressure equal to about 4000 GPa.

The above analysis shows that at the moment currently in the interior of the Jupiter-type planets, the superconducting state cannot be induced because of too high temperatures prevailing there. Nevertheless, with the passage of time, the temperature inside the planets will slowly undergo a reduction, which in turn will lead to the induction of the superconducting state in hydrogen. The theoretical predictions suggest that it will happen in the period preceding the thermal death of the Universe, when the effective temperature of the planets drops to a few Kelvin [31].

In this paper, we have analyzed the process of the induction of the superconducting state in the hydrogen layers of the Jupiter-type planet. The predictions have been based on the thermodynamic properties of hydrogen in the pressure range from ~400 GPa to 3500 GPa.

In the considered case the critical temperature of the phase transition from the normal to the superconducting state is dependent on the pairing electron–phonon interaction and the depairing Coulomb interactions, which can very significantly lower the value of T_C.

It should be noted that in the case of extremely high pressures (except $p = 200$ GPa), the critical temperature and the remaining thermodynamic parameters have not been precisely determined in the literature. First of all, the sufficiently advanced mathematical formalism has not been used and a wide range of the depairing Coulomb interactions has not been taken into account.

For this reason, in the first step of this work, the thermodynamic properties of the hydrogen layer closest to the core of the planet ($p = 3500$ GPa) have been very accurately calculated. The obtained results will allow the reader to get an idea about the properties of the superconducting state subjected to under the influence of the extremely high pressure. It should be noted that this layer is characterized by, inter alia, one of the highest critical temperatures.

The values of T_C have been also determined for the cases of the pressures at: 1000 GPa, 1500 GPa, and 2500 GPa.

2. The properties of the superconducting state close to the pressure of the planet core

2.1. The formalism

To determine the thermodynamic properties of the superconducting phase in hydrogen at the pressure at 3500 GPa, we have used the Eliashberg equations defined on the imaginary axis and in the mixed representation (both on the imaginary and the real axis) [32,33].

Let us notice that the Eliashberg equations can be derived in the framework of the microscopic theory, which is based on the Fröhlich [34]. From the physical point of view, the Fröhlich operator models the pairing interaction that leads to the formation of the superconducting state in hydrogen [16]. In particular, the linear coupling between the conducting electrons and the lattice vibrations (phonons) has been taken into account.

It should be very clearly underlined that the Eliashberg equations allow the quantitative description of the properties of the superconducting state in hydrogen [35].

The Eliashberg equations on the imaginary axis take the following form:

$$\phi_m = \frac{\pi}{\beta}\sum_{n=-M}^{M}\frac{\lambda(i\omega_m - i\omega_n) - \mu^{\star}\theta(\omega_c - |\omega_n|)}{\sqrt{\omega_n^2 Z_n^2 + \phi_n^2}}\phi_n, \tag{1}$$

$$Z_m = 1 + \frac{1}{\omega_m}\frac{\pi}{\beta}\sum_{n=-M}^{M}\frac{\lambda(i\omega_m - i\omega_n)}{\sqrt{\omega_n^2 Z_n^2 + \phi_n^2}}\omega_n Z_n, \tag{2}$$

where $M = 1100$.

The functions $\phi_m \equiv \phi(i\omega_m)$ and $Z_m \equiv Z(i\omega_m)$ represent the order parameter function and the wave function renormalization factor, respectively. The m-th Matsubara frequency is given by: $\omega_m \equiv (\pi/\beta)(2m-1)$, where $\beta \equiv (k_B T)^{-1}$ (k_B denotes the Boltzmann constant). The order parameter is defined by the expression: $\Delta_m \equiv \phi_m / Z_m$.

The electron–phonon interaction determines the values of the pairing kernel: $\lambda(z) \equiv 2\int_0^{\Omega_{\max}} d\Omega \frac{\Omega}{\Omega^2 - z^2}\alpha^2 F(\Omega)$, where $\alpha^2 F(\Omega)$ is the